Electronic Supplementary Material (ESI) for Energy & Environmental Science.

Supplementary Information

A Theoretical Study of Grating Structured Triboelectric Nanogenerators

Simiao Niu, [‡,a] Sihong Wang, [‡,a] Ying Liu, [‡,a] Yu Sheng Zhou,[a] Long Lin,[a] Youfan Hu,[a] Ken C. Pradel,[a] and Zhong Lin Wang[a,b,*]

1. Experimental verification of the FEM models

As a verification of the assumption of the FEM models, an experiment for the grating electrode case of n = 2 is performed under constant velocity motion process and comparison of the theoretical results is shown in the following figures.

Table S1. Parameters utilized in experiments and corresponding theoretical calculations

Structure Component	Parameter Utilized
Dielectric 1 (PTFE)	$\varepsilon_{r1} = 2.1$, $d_1 = 75$ μm
Dielectric 2 (Al)	$d_2 = 0$ μm
Width of Dielectrics w	3 inch
Total Length of the top plate L	4 inch
Tribo-charge surface density σ	60 μCm^{-2}
Velocity v	0.02 ms^{-1}
Moving distance x_{max}	7.5 cm

The theoretical results are generated from the method described in Section 2.2 in the manuscript.

From the comparison shown in Fig. 2, a good agreement has been reached between theoretical and experimental results, so the assumption in our model has been validated.

The error may originate from the following reasons. First, in the experiment, it is very difficult to ensure that the top plate is absolutely parallel to the bottom plate. Second, the measurement system has a signal filter and amplifier, which may filter out the high frequency component of the output signal. Third, parasitic capacitance may exist in the measurement system.

2. Calculation of SC charge for equal-length TENGs of both of grating electrodes and plate electrodes at ideal condition

2.1 SC charge for equal-length TENGs of the grating electrodes

For equal-length TENGs, since when $x = 0$, the two surfaces with the same size are fully overlapped. Therefore, $Q_0 = 0$.

As shown in the Fig. S1, the top electrodes only contains two types of regions.

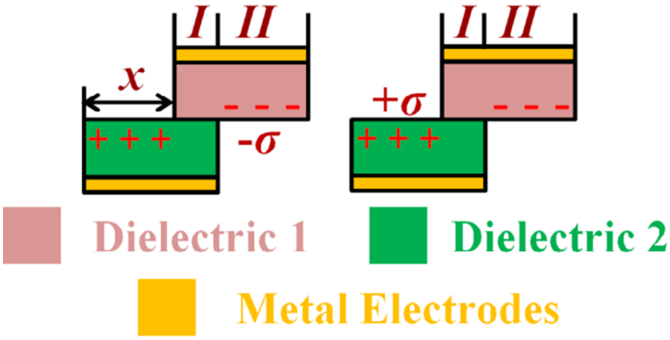

Fig. S1. Models for the grating TENGs with grating electrodes and $n = 2$.

The first type of region (Region I) is the region in which both Dielectric 1 and Dielectric 2 are existed. The second type of region (Region II) is the region in which only Dielectric 1 are existed, as shown in the following figure.

At ideal condition, since the thickness of Dielectric 1 and Dielectric 2 is thin enough, the charges distribution at each electrode region is uniform. In Region I, we assume at SC condition, the charge density at the inner surface of the top electrode is $\sigma_{I,T}$ and that at the inner surface of the top electrode is $\sigma_{I,B}$. At this region, since the bottom surface of Dielectric 1 and top surface of Dielectric 2 coincide, the surface charge density at the coincide region is the combination of Dielectric 1 and Dielectric 2, which is 0.

Since the electric field inside the electrodes are always 0, $\sigma_{I,T}$ and $\sigma_{I,B}$ satisfy the following equation.

$$\sigma_{I,B} + \sigma_{I,T} = 0 \qquad \text{(S1)}$$

At SC condition, the voltage between the top electrode and the bottom electrode is 0. Therefore, the following equation can be obtained.

$$\sigma_{I,B}\frac{d_2}{\varepsilon_0\varepsilon_{r2}} = \sigma_{I,T}\frac{d_1}{\varepsilon_0\varepsilon_{r1}} \qquad \text{(S2)}$$

Therefore, from the above the two equations, we can obtain:

$$\sigma_{I,T} = 0 \qquad \text{(S3)}$$

For Region II, the surface charge density of the top electrodes ($\sigma_{II,T}$) can be derived through the zero electric field inside the metal, which can be given by:

$$\sigma_{II,T} = \sigma \qquad \text{(S4)}$$

In step 1 ($0 \leq x \leq l$), the total length of Region I is $2(l\text{-}x)$ and the total length of Region II is $2x$. Therefore, the total charges $Q_{net,SC}$ on the top electrodes can be given by:

$$Q_{net,SC} = \sigma_{II,T}2wx + \sigma_{I,T}2w(l - x) = 2wx\sigma \qquad \text{(S5)}$$

Therefore, Q_{SC} is the difference between $Q_{net,SC}$ and Q_0, which can be given by:

$$Q_{SC} = 2wx\sigma \qquad \text{(S6)}$$

At other steps, the total length of Region I and Region II is different. With similar methods, we can derive the Q_{SC} at other steps, which is shown in Table 2 in the manuscript.

2.2 SC charge for equal-length TENGs of the plate electrodes

For equal-length TENGs with plate electrodes, there are seven different types of regions which is divided below.

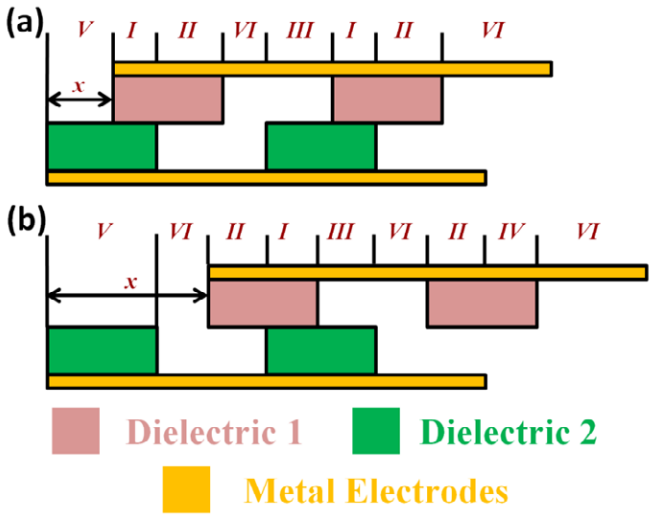

Fig. S2. Models for the grating TENGs with plate electrodes and n = 2.

Region I: Dielectric 1, Dielectric 2, top electrodes, and bottom electrodes are existed.

Region II: Dielectric 1, top electrodes, and bottom electrodes are existed.

Region III: Dielectric 2, top electrodes, and bottom electrodes are existed.

Region IV: Dielectric 1 and top electrodes are existed.

Region V: Dielectric 2 and bottom electrodes are existed.

Region VI: No dielectrics are existed.

In region I, utilizing the conditions that electric field at metal electrodes is 0 and the voltage between the top and bottom electrodes is 0, we can obtain the following equations, in which $\sigma_{A,T}$ and $\sigma_{A,B}$ represent the charge density at the inner surface of top electrodes and bottom electrodes in Region A, respectively.

$$\sigma_{I,B} + \sigma_{I,T} = 0 \qquad \text{(S7)}$$

$$\sigma_{I,B}\frac{d_2}{\varepsilon_0\varepsilon_{r2}} = \sigma_{I,T}\frac{d_1}{\varepsilon_0\varepsilon_{r1}} \qquad \text{(S8)}$$

Therefore,

$$\sigma_{I,B} = 0 \quad \text{(S9)} \qquad \sigma_{I,T} = 0 \quad \text{(S10)}$$

Similarly, for the other regions, we can obtain

$$\sigma_{II,B} = \frac{\frac{d_1}{\varepsilon_{r1}}}{\frac{d_1}{\varepsilon_{r1}} + d_2}\sigma \quad \text{(S11)} \qquad \sigma_{II,T} = \frac{d_2}{\frac{d_1}{\varepsilon_{r1}} + d_2}\sigma \quad \text{(S12)}$$

$$\sigma_{III,B} == -\frac{d_1}{d_1 + \frac{d_2}{\varepsilon_{r2}}}\sigma \quad \text{(S13)} \qquad \sigma_{III,T} = -\frac{\frac{d_2}{\varepsilon_{r2}}}{d_1 + \frac{d_2}{\varepsilon_{r2}}}\sigma \quad \text{(S14)}$$

$$\sigma_{IV,B} = 0 \quad \text{(S15)} \qquad \sigma_{IV,T} = \sigma \quad \text{(S16)}$$

$$\sigma_{V,B} = -\sigma \quad \text{(S17)} \qquad \sigma_{V,T} = 0 \quad \text{(S18)}$$

$$\sigma_{VI,B} = 0 \quad \text{(S19)} \qquad \sigma_{VI,T} = 0 \quad \text{(S20)}$$

In step 1, the length of Region II, III, and IV is $2x$, x, and 0, respectively. Therefore, the total charges $Q_{net,SC}$ on the top electrodes can be given by:

$$Q_{net,SC} = \sigma_{II,T}2wx + \sigma_{III,T}wx = \sigma wx\left(\frac{2d_2}{\frac{d_1}{\varepsilon_{r1}} + d_2} - \frac{\frac{d_2}{\varepsilon_{r2}}}{d_1 + \frac{d_2}{\varepsilon_{r2}}}\right) \qquad \text{(S21)}$$

Therefore, Q_{SC} is the difference between $Q_{net,SC}$ and Q_0, which can be given by:

$$Q_{SC} = \sigma wx\left(\frac{2d_2}{\frac{d_1}{\varepsilon_{r1}} + d_2} - \frac{\frac{d_2}{\varepsilon_{r2}}}{d_1 + \frac{d_2}{\varepsilon_{r2}}}\right) \qquad \text{(S22)}$$

Compare to the grating electrodes, the decrease Q_{SC} of is mainly because of the charge density in Region II which is less than σ and the negative charge density presented at Region III.

At other steps, the total length of each region is different. With similar methods, we can derive the Q_{SC} at other steps, which is shown in Table 2 in the text.

2.3 Capacitance trend comparison for the two electrode structure

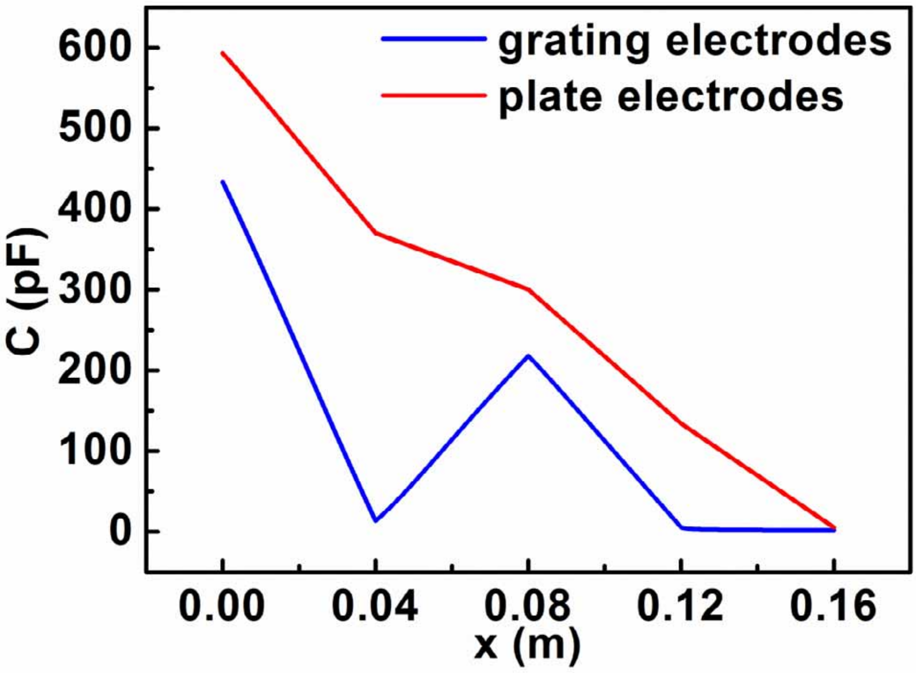

Fig. S3 Calculated capacitance trend for grating TENGs with both grating electrodes and plate electrodes.

3. Detailed interpolation procedure and results for the equal-length TENGs of $n = 4$

3.1 Extracted V_{OC} and C for different x.

Table S1. Extracted V_{OC} and C for different x FEM calculation results of equal-length TENGs of $n = 4$

x (m)	V_{OC} (V)	$1/C$ (F^{-1})
0	0.00000	2.288907E+09
0.004	28.39440	2.784648E+09
0.008	77.38480	3.641222E+09
0.012	170.30654	5.264738E+09
0.016	411.45019	9.472555E+09
0.02	1936.25100	3.591619E+10
0.024	566.19534	1.220230E+10
0.028	263.72769	6.917203E+09
0.032	143.61590	4.814789E+09
0.036	79.61060	3.693442E+09
0.04	42.39225	3.042671E+09
0.044	80.02204	3.704056E+09
0.048	144.85136	4.843445E+09
0.052	267.71757	7.001471E+09
0.056	586.37647	1.259335E+10
0.06	2659.59273	4.882669E+10
0.064	888.52206	1.793885E+10
0.068	452.25370	1.027301E+10
0.072	276.26914	7.176506E+09
0.076	181.92273	5.515364E+09
0.08	126.98063	4.549592E+09
0.084	183.02956	5.540221E+09
0.088	279.46577	7.243930E+09
0.092	462.14662	1.046993E+10
0.096	936.33534	1.883974E+10
0.1	4239.65156	7.703037E+10
0.104	1791.06418	3.400603E+10
0.108	998.32501	1.999539E+10
0.112	665.32706	1.410452E+10
0.116	483.90440	1.089367E+10
0.12	377.75310	9.017303E+09

0.124	488.52860	1.098569E+10
0.128	678.31472	1.435572E+10
0.132	1037.33964	2.072991E+10
0.136	1971.17669	3.730866E+10
0.14	10032.55395	1.804303E+11
0.144	17800.17930	3.189838E+11
0.148	20352.38664	3.645581E+11
0.152	22142.67017	3.965273E+11
0.156	23575.40302	4.221118E+11
0.16	24798.66731	4.439557E+11

3.2 Interpolation through continuous fraction method to obtain the function in the whole x region

A typical continuous fraction method interpolation function can be shown as:

$$\frac{1}{C(x)} = A(x) = a_1 + \cfrac{x - x_1}{a_2 + \cfrac{x - x_2}{a_3 + \cdots + \cfrac{x - x_4}{a_5}}} \qquad \text{(S23)}$$

$$V_{OC}(x) = b_1 + \cfrac{x - x_1}{b_2 + \cfrac{x - x_2}{b_3 + \cdots + \cfrac{x - x_4}{b_5}}} \qquad \text{(S24)}$$

Since the boundary condition changes for different half-cycles, we need to the continuous fraction for every half-cycle. Then total 8 interpolation are performed, corresponding to x at the region of 0 – 0.02, 0.02 – 0.04, 0.04 – 0.06, 0.06 – 0.08, 0.08 – 0.1, 0.1 – 0.12, 0.12 – 0.14, and 0.14 – 0.16. As an example, the interpolation parameters for x between 0 and 0.02 are listed below.

j	x_j (m)	a_j	b_j
1	0	2288906577.14882	0.00000E+00
2	0.004	8.06872950931291E-12	1.40873E-04
3	0.008	-1857917636.87880	-1.06686E+02
4	0.016	-4.07400034795834E-11	-7.28751E-04
5	0.02	-2372248789.97096	-1.48316E+02

4. Total harvested energy calculation for an equal-length grating TENG with a back and forth movement.

To calculate the total harvested energy, we need to calculate the motion process stopped at the $x = 0$. Therefore, we choose moving back and forth at constant velocity v as the motion process, which is shown below:

$$x = vt \quad \left(t < \frac{x_{max}}{v}\right) \tag{S25a}$$

$$x = 2x_{max} - vt \quad \left(\frac{2x_{max}}{v} \geq t \geq \frac{x_{max}}{v}\right) \tag{S25b}$$

$$x = 0 \quad \left(t \geq \frac{2x_{max}}{v}\right) \tag{S25c}$$

At this motion process, we can perform the numerical calculation and solve the current and voltage between $t = 0$ and $t = 2x_{max}/v$. Then we can calculate the harvested energy through this time period, which is E_1.

When t exceeds $2x_{max}/v$, x equals to 0. Therefore, the Equation 4 can be simplified as

$$R\frac{dQ}{dt} = -\frac{1}{C(x=0)}Q + V_{OC}(x=0) \tag{S26}$$

Since V_{OC} is 0 when x equals to 0, the above equation can be further simplified to:

$$R\frac{dQ}{dt} = -\frac{1}{C(x=0)}Q \tag{S27}$$

Therefore, Q when t exceeds $2x_{max}/v$ can be shown as:

$$Q = Q(t = \frac{2x_{max}}{v}) \exp\left[-\frac{1}{RC(x=0)}\left(t - \frac{2x_{max}}{v}\right)\right] \tag{S28}$$

Then the current I can be given by:

$$I = -\frac{1}{RC(x=0)}Q(t = \frac{2x_{max}}{v}) \exp\left[-\frac{1}{RC(x=0)}\left(t - \frac{2x_{max}}{v}\right)\right] \tag{S29}$$

The current decays exponentially with time. And the decay time constant τ is given by:

$$\tau = RC(x = 0) \qquad (S30)$$

When R is small, the current decays at a fast speed. While when R is sufficiently large, the decay time constant τ is sufficiently large and the current will decay at a fairly low speed.

So the energy (E_2) generated when t exceeds $2x_{max}/v$ can be given by:

$$E_2 = \int_{\frac{2x_{max}}{v}}^{+\infty} I^2 R dt = \frac{Q(t = \frac{2x_{max}}{v})^2}{2C(x = 0)} \qquad (S31)$$

The energy E_2 is just the residue electrical energy stored at the capacitance at $t = 2x_{max}/v$.

The total energy harvested in the whole process can be given by:

$$E = E_1 + E_2 = \int_0^{\frac{2x_{max}}{v}} I^2 R dt + \frac{Q(t = \frac{2x_{max}}{v})^2}{2C(x = 0)} \qquad (S32)$$

The energy curves shown in Fig. 3d are obtained through Equation S32.

5. Q_{SC} and η_{CT} for the unequal-length grating TENG at ideal condition

For simplicity, we mainly discuss Q_{SC} and η_{CT} for the first half period. Q_{SC} and η_{CT} for the other period can be obtained through the symmetry and periodicity.

As shown in the Fig. S4, the top electrodes only contains two types of regions.

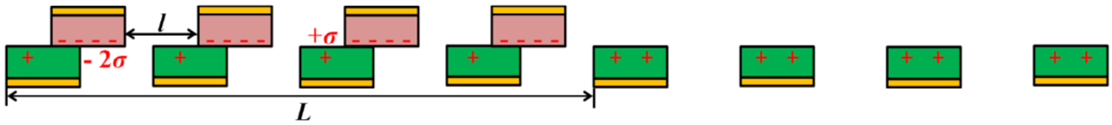

Fig. S4. Models for the unequal-length grating TENGs with plate electrodes and n = 4.

The first type of region is the region in which both Dielectric 1 and Dielectric 2 are existed, as shown in the following figure.

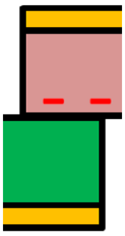

Fig. S5. Models for the first type of region for the unequal-length grating TENGs with grating electrodes.

The second type of region is the region in which only Dielectric 1 are existed, as shown in the following figure.

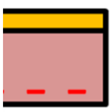

Fig. S6. Models for the second type of region for the unequal-length grating TENGs with grating electrodes.

At ideal condition, since the thickness of Dielectric 1 and Dielectric 2 is thin enough (the condition of ideal approximation), the charges distribution at each electrode region is uniform. In the first type of region, we assume at SC condition, the charge density at the inner surface of the top electrode is σ_{AT} and that at the inner surface of the top electrode is σ_{AB}. At this region, since the bottom surface of Dielectric 1 and top surface of Dielectric 2 coincide, the surface charge density at the coincide region is the combination of Dielectric 1 and Dielectric 2, which is $-\sigma$.

Since the electric field inside the electrodes are always 0, σ_{AT} and σ_{AB} satisfy the following equation.

$$\sigma_{AB} + \sigma_{AT} = \sigma \qquad \text{(S33)}$$

At SC condition, the voltage between the top electrode and the bottom electrode is 0. Therefore, the following equation can be obtained.

$$\sigma_{AB}\frac{d_2}{\varepsilon_0\varepsilon_{r2}} = \sigma_{AT}\frac{d_1}{\varepsilon_0\varepsilon_{r1}} \qquad \text{(S34)}$$

Therefore, from the above the two equations, we can obtain:

$$\sigma_{AT} = \frac{\frac{d_2}{\varepsilon_{r2}}}{\frac{d_2}{\varepsilon_{r2}} + \frac{d_1}{\varepsilon_{r1}}}\sigma = \frac{1}{1 + \frac{d_1\varepsilon_{r2}}{d_2\varepsilon_{r1}}}\sigma \qquad \text{(S35)}$$

For the second type of region, the surface charge density of the top electrodes can be derived through the zero electric field inside the metal, which can be given by:

$$\sigma_{BT} = 2\sigma \qquad \text{(S36)}$$

The total length of the first type of region is $n(l\text{-}x)$ and the total length of the second type of region is nx. Therefore, the total charges $Q_{net,SC}$ on the top electrodes can be given by:

$$Q_{net,SC} = \sigma_{BT}nwx + \sigma_{AT}nw(l - x) \qquad \text{(S37)}$$

Since Q_0 is defined as the net charges at SC condition when $x = 0$. Therefore, Q_0 can be derived through the following equation, which is a non-zero value.

$$Q_0 = Q_{net-to,SC}(x = 0) = \sigma_{AT} nwl \tag{S38}$$

Therefore, Q_{SC} is the difference between $Q_{net,SC}$ and Q_0, which can be given by:

$$Q_{SC} = (\sigma_{BT} - \sigma_{AT}) nwx = \left(2 - \frac{1}{1 + \frac{d_1 \varepsilon_{r2}}{d_2 \varepsilon_{r1}}} \right) n\sigma wx \tag{S39}$$

6. Period boundary condition for the unequal-length grating TENG at constant velocity motion process

6.1 Numerical calculation shows that the converge to the steady periodic curve

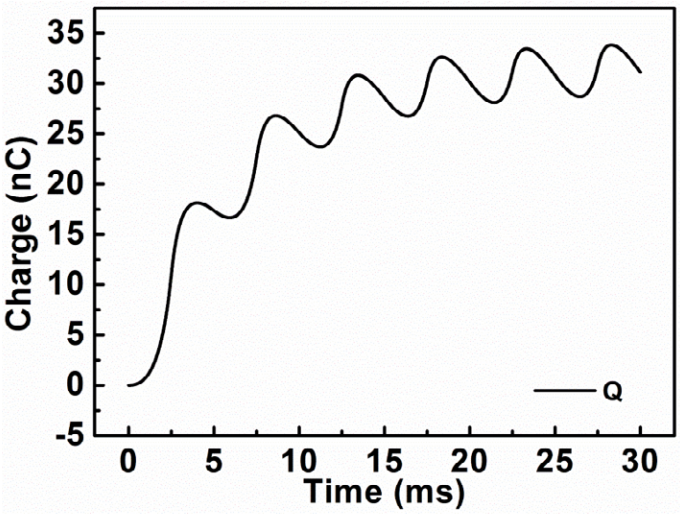

Fig. S7. Charge-time curve for unequal-length grating TENGs at constant velocity and $Q(t = 0) = 0$ boundary condition.

6.2 Derivation for Equation 15

We have Equation 4 as the basic differential equation for the load performance, which is shown as below:

$$R\frac{dQ}{dt} = -\frac{1}{C} \times Q + V_{OC} \quad \text{(S40)}$$

Considering the first half cycle between $t = 0$ and $t = l/v$, the solution for the above differential equation with the boundary condition $Q(t = 0) = Q_0$ at constant velocity motion can be given by:

$$Q = \exp\left(-\int_0^t \frac{dt}{C(vt)R}\right)\left[Q_0 + \int_0^t \frac{V_{OC}(vt)}{R} \exp\left(\int_0^t \frac{1}{C(vz)R} dz\right) dt\right]$$

$$= \exp\left(-\frac{1}{Rv}\int_0^{vt}\frac{dx}{C(x)}\right)\left[Q_0 + \int_0^{vt}\frac{V_{OC}(x)}{Rv}\exp\left(\frac{1}{Rv}\int_0^{x}\frac{dz}{C(z)}\right)dx\right] \quad \text{(S41)}$$

Therefore, at the end of this half cycle, Q is given by:

$$Q_1 = Q(t = \tfrac{l}{v}) = \exp\left(-\tfrac{1}{Rv}\int_0^{l}\tfrac{dx}{C(x)}\right)\left[Q_0 + \tfrac{1}{Rv}\int_0^{l}V_{OC}(x)\exp\left(\tfrac{1}{Rv}\int_0^{x}\tfrac{dz}{C(z)}\right)dx\right] \quad \text{(S42)}$$

At the second half cycle between $t = l/v$ and $t = 2l/v$, the solution for the above the differential equation with the boundary condition $Q(t = l/v) = Q_1$ at constant velocity motion can be given by:

$$Q = \exp\left(-\int_{\frac{l}{v}}^{t}\frac{dt}{C(vt)R}\right)\left[Q_1 + \int_{\frac{l}{v}}^{t}\frac{V_{OC}(vt)}{R}\exp\left(\int_{\frac{l}{v}}^{t}\frac{1}{C(vz)R}dz\right)dt\right]$$

$$= \exp\left(-\frac{1}{Rv}\int_l^{vt}\frac{dx}{C(x)}\right)\left[Q_1 + \int_l^{vt}\frac{V_{OC}(x)}{Rv}\exp\left(\frac{1}{Rv}\int_l^{x}\frac{dz}{C(z)}\right)dx\right] \quad \text{(S43)}$$

Therefore, at the end of this second half cycle, Q is given by:

$$Q_2 = Q(t = \frac{2l}{v}) = \exp\left(-\frac{1}{Rv}\int_l^{2l}\frac{dx}{C(x)}\right)\left[Q_1 + \int_l^{2l}\frac{V_{OC}(x)}{Rv}\exp\left(\frac{1}{Rv}\int_l^{x}\frac{dz}{C(z)}\right)dx\right]$$

$$= \exp\left(-\frac{1}{Rv}\int_0^{l}\frac{dx}{C(2l-x)}\right)\left[Q_1 + \int_0^{l}\frac{V_{OC}(2l-x)}{Rv}\exp\left(\frac{1}{Rv}\int_l^{2l-x}\frac{dz}{C(z)}\right)dx\right]$$

$$= \exp\left(-\frac{1}{Rv}\int_0^{l}\frac{dx}{C(2l-x)}\right)\left[Q_1 + \int_0^{l}\frac{V_{OC}(2l-x)}{Rv}\exp\left(\frac{1}{Rv}\int_x^{l}\frac{dz}{C(2l-z)}\right)dx\right] \quad \text{(S44)}$$

From Equation 11, the above equation can be further simplified to:

$$Q_2 = \exp\left(-\frac{1}{Rv}\int_0^{l}\frac{dx}{C(x)}\right)\left[Q_1 + \int_0^{l}\frac{V_{OC}(x)}{Rv}\exp\left(\frac{1}{Rv}\int_x^{l}\frac{dz}{C(z)}\right)dx\right]$$

$$= \exp\left(-\frac{1}{Rv}\int_0^{l}\frac{dx}{C(x)}\right)\left[Q_1 + \int_0^{l}\frac{V_{OC}(x)}{Rv}\exp\left(\frac{1}{Rv}\int_0^{l}\frac{dz}{C(z)} - \frac{1}{Rv}\int_0^{x}\frac{dz}{C(z)}\right)dx\right]$$

$$= \exp\left(-\frac{1}{Rv}\int_0^{l}\frac{dx}{C(x)}\right)Q_1 + \frac{1}{Rv}\int_0^{l}V_{OC}(x)\exp\left(-\frac{1}{Rv}\int_0^{x}\frac{dz}{C(z)}\right)dx \quad \text{(S45)}$$

The period boundary condition is applied. Therefore,

$$Q_0 = Q_2 = \exp\left(-\frac{1}{Rv}\int_0^l \frac{dx}{C(x)}\right) Q_1 + \frac{1}{Rv}\int_0^l V_{OC}(x)\exp\left(-\frac{1}{Rv}\int_0^x \frac{dz}{C(z)}\right) dx \quad \text{(S46)}$$

Combining Equation S42 and S46, we can obtain the equation for Q_0, which is the same as Equation 15.

$$Q_0 = \frac{\exp\left(-\frac{2}{Rv}\int_0^l \frac{1}{C(x)} dx\right)}{1-\exp\left(-\frac{2}{Rv}\int_0^l \frac{1}{C(x)} dx\right)} \frac{1}{Rv}\int_0^l V_{OC}(x)\exp\left(\frac{1}{Rv}\int_0^x \frac{1}{C(z)} dz\right) dx$$

$$+\frac{1}{1-\exp\left(-\frac{2}{Rv}\int_0^l \frac{1}{C(x)} dx\right)} \frac{1}{Rv}\int_0^l V_{OC}(x)\exp\left(-\frac{1}{Rv}\int_0^x \frac{1}{C(z)} dz\right) dx \quad \text{(S47)}$$

Besides, we still want to discuss two special cases of this period boundary condition, which is SC and OC condition.

At SC condition, *R* is 0. From the above equation, the period boundary condition can be shown as:

$$Q_0(R=0) = 0 \quad \text{(S48)}$$

At OC condition, *R* is infinity. At this condition, we have the following first-order approximation.

$$\exp\left(-\frac{2}{Rv}\int_0^l \frac{1}{C(x)} dx\right) \approx 1 - \frac{2}{Rv}\int_0^l \frac{1}{C(x)} dx \quad \text{(S49)}$$

Therefore, the above equation can be simplified to:

$$Q_0 = \frac{1 - \frac{2}{Rv}\int_0^l \frac{1}{C(x)}dx}{\frac{2}{Rv}\int_0^l \frac{1}{C(x)}dx} \frac{1}{Rv}\int_0^l V_{OC}(x)\left(1 + \frac{1}{Rv}\int_0^x \frac{1}{C(z)}dz\right)dx$$

$$+ \frac{1}{\frac{2}{Rv}\int_0^l \frac{1}{C(x)}dx} \frac{1}{Rv}\int_0^l V_{OC}(x)\left(1 - \frac{1}{Rv}\int_0^x \frac{1}{C(z)}dz\right)dx \quad \text{(S50)}$$

$$Q_0 = \frac{1 - \frac{2}{Rv}\int_0^l \frac{1}{C(x)}dx}{2\int_0^l \frac{1}{C(x)}dx} \int_0^l V_{OC}(x)\left(1 + \frac{1}{Rv}\int_0^x \frac{1}{C(z)}dz\right)dx$$

$$+ \frac{1}{2\int_0^l \frac{1}{C(x)}dx} \int_0^l V_{OC}(x)\left(1 - \frac{1}{Rv}\int_0^x \frac{1}{C(z)}dz\right)dx \quad \text{(S51)}$$

When R is close to infinity,

$$Q_0(R = +\infty) = \frac{\int_0^l V_{OC}(x)dx}{\int_0^l \frac{1}{C(x)}dx} \quad \text{(S52)}$$

The $Q_0 - R$ curve is shown below.

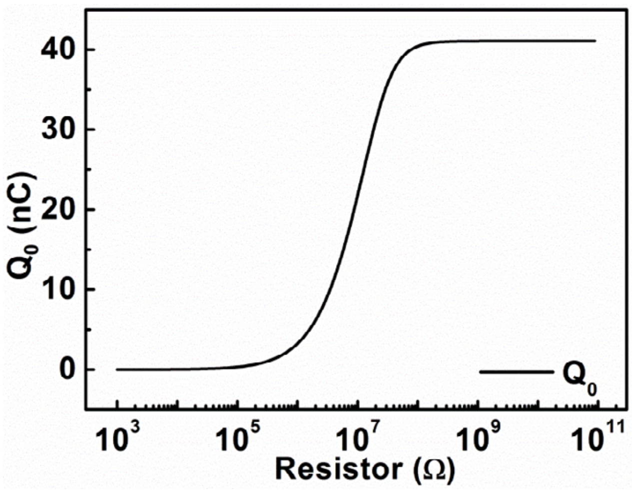

Fig. S8. Periodic boundary condition for unequal-length grating TENGs at constant velocity motion process.

7. Proof of Equation 22

We first consider the period boundary condition for unequal-length grating TENG A (Q_{0A}) and B (Q_{0B}).

From Equation 15,

$$Q_{OA}$$

$$= \frac{\exp\left(-\frac{2}{R_A v}\int_0^{l_A}\frac{1}{C_A(x)}dx\right)}{1-\exp\left(-\frac{2}{R_A v}\int_0^{l_A}\frac{1}{C_A(x)}dx\right)}\frac{1}{R_A v}\int_0^{l_A}V_{OC,A}(x)\exp\left(\frac{1}{R_A v}\int_0^{x}\frac{1}{C_A(z)}dz\right)dx$$

$$+\frac{1}{1-\exp\left(-\frac{2}{R_A v}\int_0^{l_A}\frac{1}{C_A(x)}dx\right)}\frac{1}{R_A v}\int_0^{l_A}V_{OC,A}(x)\exp\left(-\frac{1}{R_A v}\int_0^{x}\frac{1}{C_A(z)}dz\right)dx \quad \text{(S53)}$$

$$Q_{OB}$$

$$= \frac{\exp\left(-\frac{2}{R_B v}\int_0^{l_B}\frac{1}{C_B(x)}dx\right)}{1-\exp\left(-\frac{2}{R_B v}\int_0^{l_B}\frac{1}{C_B(x)}dx\right)}\frac{1}{R_B v}\int_0^{l_B}V_{OC,B}(x)\exp\left(\frac{1}{R_B v}\int_0^{x}\frac{1}{C_B(z)}dz\right)dx$$

$$+\frac{1}{1-\exp\left(-\frac{2}{R_B v}\int_0^{l_B}\frac{1}{C_B(x)}dx\right)}\frac{1}{R_B v}\int_0^{l_B}V_{OC,B}(x)\exp\left(-\frac{1}{R_B v}\int_0^{x}\frac{1}{C_B(z)}dz\right)dx$$

$$= \frac{\exp\left(-\frac{2k^2}{R_A v}\int_0^{\frac{l_A}{k}}\frac{1}{kC_A(kx)}dx\right)}{1-\exp\left(-\frac{2k^2}{R_A v}\int_0^{\frac{l_A}{k}}\frac{1}{kC_A(kx)}dx\right)}\frac{k^2}{R_A v}\int_0^{\frac{l_A}{k}}\frac{1}{k}V_{OC,A}(kx)\exp\left(\frac{k^2}{R_A v}\int_0^{x}\frac{1}{kC_A(kz)}dz\right)dx$$

$$+\frac{1}{1-\exp\left(-\frac{2k^2}{R_A v}\int_0^{\frac{l_A}{k}}\frac{1}{kC_A(kx)}dx\right)}\frac{k^2}{R_A v}\int_0^{\frac{l_A}{k}}\frac{1}{k}V_{OC,A}(kx)\exp\left(-\frac{k^2}{R_A v}\int_0^{x}\frac{1}{kC_A(kz)}dz\right)dx$$

$$=\frac{\exp\left(-\frac{2}{R_A v}\int_0^{\frac{l_A}{k}}\frac{k}{C_A(kx)}dx\right)}{1-\exp\left(-\frac{2}{R_A v}\int_0^{\frac{l_A}{k}}\frac{k}{C_A(kx)}dx\right)}\frac{1}{R_A v}\int_0^{\frac{l_A}{k}} kV_{OC,A}(kx)\exp\left(\frac{1}{R_A v}\int_0^{kx}\frac{1}{C_A(z)}dz\right)dx$$

$$+\frac{1}{1-\exp\left(-\frac{2}{R_A v}\int_0^{\frac{l_A}{k}}\frac{k}{C_A(kx)}dx\right)}\frac{1}{R_A v}\int_0^{\frac{l_A}{k}} kV_{OC,A}(kx)\exp\left(-\frac{1}{R_A v}\int_0^{kx}\frac{1}{C_A(z)}dz\right)dx$$

$$=\frac{\exp\left(-\frac{2}{R_A v}\int_0^{l_A}\frac{1}{C_A(x)}dx\right)}{1-\exp\left(-\frac{2}{R_A v}\int_0^{l_A}\frac{1}{C_A(x)}dx\right)}\frac{1}{R_A v}\int_0^{l_A} V_{OC,A}(x)\exp\left(\frac{1}{R_A v}\int_0^{x}\frac{1}{C_A(z)}dz\right)dx$$

$$+\frac{1}{1-\exp\left(-\frac{2}{R_A v}\int_0^{l_A}\frac{1}{C_A(x)}dx\right)}\frac{1}{R_A v}\int_0^{l_A} V_{OC,A}(x)\exp\left(-\frac{1}{R_A v}\int_0^{x}\frac{1}{C_A(z)}dz\right)dx$$

$$=Q_{OA} \qquad \text{(S54)}$$

Therefore, we have proven that Q_{0A} equals to Q_{0B}.

Then we are going to prove Equation 22. For the first half cycle, solving Equation 4, we can obtain:

$$Q_A(t)=\exp\left(-\int_0^t\frac{dz}{C_A(vz)R_A}\right)\left[Q_{OA}+\int_0^t\frac{V_{OC,A}(vz)}{R_A}\exp\left(\int_0^z\frac{1}{C_A(vz)R_A}dz\right)dz\right] \qquad \text{(S55)}$$

$$Q_B(t)=\exp\left(-\int_0^t\frac{dz}{C_B(vz)R_B}\right)\left[Q_{OB}+\int_0^t\frac{V_{OC,B}(vz)}{R_B}\exp\left(\int_0^z\frac{1}{C_B(vz)R_B}dz\right)dz\right]$$

$$=\exp\left(-\int_0^t\frac{k^2dz}{kC_A(kvz)R_A}\right)\left[Q_{OA}+\int_0^t\frac{kV_{OC,A}(kvz)}{R_A}\exp\left(\int_0^z\frac{k^2}{kC_A(kvz)R_A}dz\right)dz\right]$$

$$=\exp\left(-\int_0^t\frac{kdz}{C_A(kvz)R_A}\right)\left[Q_{OA}+\int_0^t\frac{kV_{OC,A}(kvz)}{R_A}\exp\left(\int_0^z\frac{k^2}{kC_A(kvz)R_A}dz\right)dz\right]$$

$$= \exp\left(-\int_0^{kt} \frac{dz}{C_A(vz)R_A}\right)\left[Q_{OA} + \int_0^{kt} \frac{V_{OC,A}(vz)}{R_A} \exp\left(\int_0^{z} \frac{1}{C_A(vz)R_A} dz\right) dz\right]$$

$$= Q_A(kt) \quad \text{(S56)}$$

8. Influence of L on the optimum grating ratio at constant velocity motion process

Consider two unequal-length grating TENG (C and D) with Dielectric 1 thickness of L_C and L_D ($L_D = k^* L_C$). To maintain the same aspect ratio, the following equation is satisfied.

$$n_D = kn_C \quad \text{(S57)} \qquad l_C = l_D \quad \text{(S58)}$$

Similarly, the V_{OC} and C of TENG C and D have the following relationship.

$$V_{OC,C}(x) = V_{OC,D}(x) \quad (0 \leq x \leq l) \quad \text{(S59)} \quad C_D(x) = kC_C(x) \quad (0 \leq x \leq l) \quad \text{(S60)}$$

Thus, similarly, we can prove that the total energy generated from TENG C at the load resistance R_C and TENG D at the load resistance $R_D = R_C/k$ satisfies the following equation.

$$E_D(x) = kE_C(x) \qquad \text{(S61)}$$

Thus, the optimum resistance of grating TENG C is k times larger than that of grating TENG D. Moreover, when the TENG C and TENG D both have their optimum load, their maximum total harvested energy is has this ratio as well. Since for any arbitrary aspect ratio, TENG C and TENG D have the same ratio considering the maximum harvested energy, the optimum grating aspect ratio is independent on L in this motion process.

9. Influence of v on the optimum grating ratio at constant velocity motion process

Consider the same unequal-length grating TENG under two different moving and load conditions: moving at velocity v_E with load resistance R_E (Condition 1) and moving at velocity v_F with load resistance R_F (Condition 2), satisfying that $v_E = kv_F$ and $R_F = kR_E$. Therefore, through Equation 15 in the text, the period boundary condition for this two cases are completely the same.

$$Q_{0E} = Q_{0F} \qquad \text{(S62)}$$

Similar to Equation S55, the charge-time curve for moving at velocity v_E with load resistance R_E can be given by:

$$\begin{aligned} Q_E(t) &= \exp\left(-\int_0^t \frac{dz}{C(v_E z)R_E}\right)\left[Q_{OE} + \int_0^t \frac{V_{OC}(v_E z)}{R_E}\exp\left(\int_0^z \frac{dz}{C(v_E z)R_E}\right)dz\right] \\ &= \exp\left(-\frac{1}{v_E R_E}\int_0^{v_E t}\frac{dz}{C(z)}\right)\left[Q_{OE}\right. \\ &\quad \left. + \int_0^{v_E t}\frac{V_{OC}(z)}{v_E R_E}\exp\left(\frac{1}{v_E R_E}\int_0^z \frac{dz}{C(z)}\right)dz\right] \qquad \text{(S63)} \end{aligned}$$

$$\begin{aligned} Q_F(t) &= \exp\left(-\frac{1}{v_F R_F}\int_0^{v_F t}\frac{dz}{C(z)}\right)\left[Q_{OF}\right. \\ &\quad \left. + \int_0^{v_F t}\frac{V_{OC}(z)}{v_F R_F}\exp\left(\frac{1}{v_F R_F}\int_0^z \frac{dz}{C(z)}\right)dz\right] \qquad \text{(S64)} \end{aligned}$$

Therefore,

$$Q_F(kt) = \exp\left(-\frac{1}{v_F R_F}\int_0^{kv_F t}\frac{dz}{C(z)}\right)\left[Q_{OF} + \int_0^{kv_F t}\frac{V_{OC}(z)}{v_F R_F}\exp\left(\frac{1}{v_F R_F}\int_0^z \frac{dz}{C(z)}\right)dz\right]$$

$$= \exp\left(-\frac{1}{v_E R_E}\int_0^{v_E t}\frac{dz}{C(z)}\right)\left[Q_{OE} + \int_0^{v_E t}\frac{V_{OC}(z)}{v_E R_E}\exp\left(\frac{1}{v_E R_E}\int_0^{z}\frac{dz}{C(z)}\right)dz\right]$$

$$= Q_E(t) \qquad \left(0 \le t \le \frac{l}{v_F}\right) \quad \text{(S65)}$$

Therefore,

$$I_E(t) = \frac{\mathrm{d}Q_E}{\mathrm{d}t} = kI_F(kt), \ \left(0 \le t \le \frac{l}{v_F}\right) \quad \text{(S66)}$$

$$V_E(t) = I_E(t)R_E = V_F(kt), \qquad \left(0 \le t \le \frac{l}{v_F}\right) \quad \text{(S67)}$$

The relationship for the energy generated in the whole sliding process can be given by:

$$E_E = 2n_E\int_0^{l/v_E} I_E(t)V_E(t)dt = 2kn_F\int_0^{l/(kv_F)} I_F(kt)V_F(kt)dt = 2n_F\int_0^{l/v_F} I_F(t)V_F(t)dt$$

$$= E_F \qquad \text{(S68)}$$

Therefore, the total energy harvested at Condition 1 is completely identical to that of at Condition 2. Thus, the optimum resistance for the TENG at velocity *v* is also *k* times as large as that at velocity *kv* and the optimum grating aspect ratio for each unit is independent on *v* in this constant velocity motion process.