A Revised Conversion Factor Relating Respirable Dust Concentrations Measured by 10 mm Dorr-Oliver Nylon Cyclones Operated at 1.7 and 2.0 L min^{-1}

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Appendix B

B-1 General analytical model

As a result of multiplicative errors arising from various errors associated with the personal sampler[B1], the total sum of squares in regression analysis will primarily be influenced by the large dependent variable values and lead to an analysis bias. This situation is typical of data collected with dust sampling instrumentation[B1], and there are several different remedial data transformations available to eliminate, or at least minimize, the non-constant variance problem.

Weighted regression was the method of choice previously reported[B2] to stabilize the variance for data analysis according to the model

Eq.B-1 $$Y = g(X)\varepsilon_1 + {}_0\varepsilon_0,$$

where

Y = a response variable,
$g(X) = aX$, and
a = regression slope, assuming zero (statistically insignificant or unmeasurable) intercept.

The only requirement in Eq. B-1 is that $g(X)$ be a smooth, arbitrary function of the predictor variable (X). Intuitively, one would expect, in the absence of measurement bias, a linear and monotonic relationship (ideally with zero intercept and unity slope) between different instruments designed and developed to measure the same true but unknown quantity.

Weighted regression can directly stabilize the variance if the variance function can be estimated. Typical $1/X^2$ weighting assumes that the dependent variable variance increases proportionally with X^2 over the entire range of independent variable, resulting in constant coefficient of variation (CV). However, at low concentration values there is the limiting error term (${}_0\varepsilon_0$) due to weighing imprecision.

B-2 Weight variable estimation

The constant variance $({}_0\sigma)^2$ of ${}_0\varepsilon_0$ can usually be determined with sufficient accuracy. It can readily be shown by calculating Var(Y), using the method of expectation values, that the proper weight variable is the reciprocal of the total variance σ_T^2, given by

Eq. B-2 $$\sigma_T^{\,2} = ({}_0\sigma)^2 + [(g(X)CV]^2,$$
$$= ({}_0\sigma)^2 + (aCV)^2X^{\,2},$$

where

From Eq. B-2, *CV* can be considered to be the variation of the dependent variable about the regression line.

The process for estimating the proper weight variable is iterative, using the following procedure for estimating the relationship between the dependent variable variance and the independent variable:

Step 1: An initial regression of Eq. B-1 using $1/X^{\,2}$ weighting is performed to establish initial weight variables, where $g(X) = aX + Y_0$.
Step 2: Using the definition of variance, the values $(Y_i - Y_{ip})^2$, representing the variance between the measured Y_i and predicted Y_{ip} from the initial regression of step 1, are calculated.
Step 3: The plot of $(Y_i - Y_{ip})^2$ vs. X_i is fit with the function of Eq. B-2, represented by the model

Eq. B-3 $$(Y_i - Y_{ip})^2 = c^2 + dX^{\,2}$$

using $1/X^{\,2}$ weighting. The second weight estimation is then approximated point-by-point as $1/\sigma_T^{\,2}$.
Step 4: A weighted regression is next performed with the new weight variable.
Steps 2-4 are then repeated with each new estimate of weight variable and $(Y_i - Y_{ip})^2$ until convergence to a solution. If intercept Y_0 is shown to be irrelevant due to statistical insignificance, being an unmeasurable quantity, or some other justification, steps 1-4 are repeated with $Y_0 = 0$.

References

[B1] NIOSH, *The Precision of Coal Mine Dust Sampling*. US Department of Health and Human Services, Public Health Services, Centers for Disease Control, National Institute for Occupational Safety and Health, NTIS Pub. No. PB85220721, 1984.

[B2] S. J. Page, J.C. Volkwein, R. P. Vinson, G. J. Joy, S. Mischler, D. P. Tuchman, L. J. McWilliams, *J Environ. Monit.* 2008, **10**, 96-101.