Electronic Supplementary Material (ESI) for Faraday Discussions.
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Supplementary Material

1 Statistical noise analysis

The origin of noise in the frequency domain was investigated by constructing a histogram of line intensities (heights) using data points from the power spectrum. The height distribution of noise in the frequency domain follows from the distribution in the time domain. Provided neighbouring data points in the time domain are independent of each other and intensities are distributed normally with zero mean and a variance σ^2 (white noise) the intensity distribution in the frequency domain is given by a second order $\chi^2_{k=2}$ distribution[1], defined by

$$\chi^2_{k=2}(x) = de^{-ax} \tag{1}$$

where x is the height, i.e. the occurrence of a specific intensity. When the noise comes only from white noise $d = a = 1/2$. An exponential function $y = de^{-ax}$ was fitted to the histogram of intensities and the parameters a and d were determined to test whether the noise observed in the frequency domain originates from white noise in the time-domain.

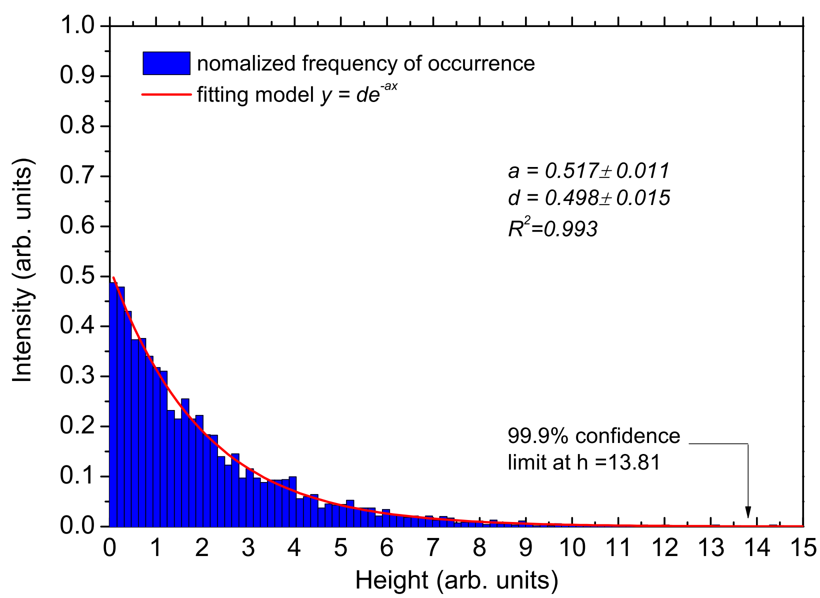

Fig. 1 A histogram of line intensities (see text for more details). The distribution was normalised and centred at the mean of $\mu = 2$. The function $y = de^{-ax}$ was fitted and the best estimates of the coefficients, $a = 0.517 \pm 0.011$ and $d = 0.498 \pm 0.015$, are in good agreement with the expected coefficients of a $\chi^2_{k=2}$ distribution. The goodness of fit was confirmed through the R^2 parameter.

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The histogram of intensities was generated by sorting 4700 data points into 100 equal-sized bins. The resulting distribution of intensities was then scaled in order to fulfil the requirement of a mean, $\mu = 2$, for a $\chi^2_{k=2}$ distribution as well as normalised by the total area under the curve. The resulting histogram is shown in Fig. 1. The parameters of the exponential function $y = de^{-ax}$ were determined from a linear least squares analysis. The extracted values, $a = 0.517 \pm 0.011$ and $d = 0.498 \pm 0.015$, are in excellent agreement with the expected coefficients of 0.500 and 0.500, respectively, which is also reflected in a goodness-of-fit value[2], $R^2 = 0.993$. It was therefore concluded that the intensity variations seen in the high frequency part in the spectrum are due to white noise.

The 99.9% significance level for the random events in the spectrum was established by finding the intensity level for which the following integral equation holds:

$$\frac{1}{2}\int_0^x e^{-s/2} ds = 0.999 \tag{2}$$

The solution to this equation is $x = 13.81$, which corresponds to an amplitude of 7.7×10^{-10} in the power spectrum. The derived confidence level was applied in the low frequency region and any signal above this limit was treated as real with a likelihood of at least 99.9 %. We note that this result is based on the assumption that other sources of noise, such as $1/f$ noise, are absent. Furthermore, repeat scans with similar and different pump laser powers show similar spectral features and were taken as additional evidence that the observed peaks are real.

References

1 M. R. Spiegel, J. J. Schiller and A. V. Srinivasan, *Schaums Easy Outline of Probability and Statistics"*, McGraw-Hill Publishing Company, 2001.

2 D. Salvatore and D. Reagle, *Schaum's Outline of Statistics and Econometrics*, McGraw-Hill, 2nd edn., 2001.

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