# SUPPLEMENTARY MATERIAL

## Fed batch mass balance

$$Accumulation = in - out + production$$

Mass balance on GABA: nothing is going in or out, except that solid glutamic acid is going in. So the accumulation of GABA is equal to the production of it:

$$V_{reactor} \frac{dC}{dt} = r_{GABA} \cdot m_{GAD} \tag{1}$$

$$\int_{C_{GABA}^{t=0}}^{C_{GABA}^{t=t}} dC = \int_{t_0}^{t_t} \frac{r_{GABA} \cdot m_{GAD}}{V_{reactor}} dt \tag{2}$$

The enzyme activity ($r_{GABA}$) can be described with the empirically determined equation for inactivation:

$$r_{GABA}(t) = r_{GABA}^{init} \cdot (0.71 \cdot e^{-0.014t} + 0.36 \cdot e^{-0.74t}) \tag{3}$$

$C_{Glu}$= 100 mM. This concentration is kept constant by the continuous addition of solid glutamic acid to the reactor. That means that the initial reaction rate is equal to $v_{max}$, which is 117 U/mg at 30 $^{o}$C and pH 4.6, equal to 7.02 mol/g.hr.

$$\int_{C_{GABA}^{t=0}}^{C_{GABA}^{t=t}} dC = \frac{r_{GABA}^{init} \cdot m_{GAD}}{V_{reactor}} \int_{t_0}^{t_t} (0.71 \cdot e^{-0.014t} + 0.36 \cdot e^{-0.74t}) dt \tag{4}$$

Because glutamic acid is added in solid form, also $V_{reactor}$ is assumed to be independent of time, so it can be removed from both sides of equation 4:

$$\int_{n_{GABA}^{t=0}}^{n_{GABA}^{t=t}} dn = r_{GABA}^{init} \cdot m_{GAD} \int_{t_0}^{t_t} (0.71 \cdot e^{-0.014t} + 0.36 \cdot e^{-0.74t}) dt \quad (5)$$

The reaction will be stopped when there is only 5% enzyme activity left, which is after 192 hours:

$$n_{GABA} = 7.02 \cdot 1 \cdot \left[ \frac{0.71}{-0.014} \cdot e^{-0.014t} + \frac{0.36}{-0.74} \cdot e^{-0.74t} \right]_{192}^{0} \quad (6)$$

$n_{GABA}$ = 137 mol/g, so the overall productivity of this bioreactor loaded with 1 g enzyme is 34 kg GABA in eight days.