# Appendix:

The one-dimensional convective-diffusion in a circular channel geometry satisfies

$$\frac{\partial c}{\partial t}+\frac{U}{R}\frac{\partial c}{\partial \theta}=\frac{D_{eff}}{R^2}\frac{\partial^2 c}{\partial \theta^2}$$

where $c$ is the concentration averaged over the cross-section, $U$ is the average velocity, $R$ is the radius of the centerline and $D_{eff}$ is the dispersivity.

The solution of this equation is obtained by letting $\theta \to \theta - \frac{U}{R}t$, which gives the usual diffusion equation. We solve this equation assuming the solution is 2π periodic and so are led to

$$c(\theta,t)=\sum_{n=0}^{\infty}\left[a_n \cos\left(n\left(\theta-\frac{U}{R}t\right)\right)+b_n \sin\left(n\left(\theta-\frac{U}{R}t\right)\right)\right]e^{-n^2 D_{eff} t/R^2}$$

with the Fourier coefficients being

$$a_0=\frac{1}{2\pi}\int_0^{2\pi} c(\theta,0)d\theta,$$

$$a_n=\frac{1}{2\pi}\int_0^{2\pi} c(\theta,0)\cos(n\theta)d\theta \quad n\neq 0$$

$$b_n=\frac{1}{2\pi}\int_0^{2\pi} c(\theta,0)\sin(n\theta)d\theta \quad n\neq 0$$

and $c(\theta,0)$ is the initial distribution of fluorescein in the rotary mixer. We choose $\theta$=0 as the position where we recorded fluorescence intensity.

At long times, $c(0,t)$ can be approximated by the first term in the series

$$c(0,t)=a_0+\left[a_1\cos\left(\frac{U}{R}t\right)-b_1\sin\left(\frac{U}{R}t\right)\right]e^{-D_{eff}t/R^2}$$

The extrema of this function are obtained for $\frac{\partial c(0,t)}{\partial t}=0$, with

$$\frac{\partial c(0,t)}{\partial t}=\left[\frac{U}{R}\left(a_1\sin\left(\frac{U}{R}t\right)+b_1\cos\left(\frac{U}{R}t\right)\right)-\frac{D_{eff}}{R^2}\left(a_1\cos\left(\frac{U}{R}t\right)-b_1\sin\left(\frac{U}{R}t\right)\right)\right]e^{-D_{eff}t/R^2}$$

As discussed, for the geometry of the rotary mixer, the mixing regime is controlled by the hydrodynamic dispersion for $U$ <7.5 $10^{-1}$mm/s. Using $k$=0.003 for channels of parabolic shape, $w$=0.1mm, R=1 mm for our devices and $D$=3*$10^{-4}$mm$^2$/s for the fluorescein, we have $D_{eff}$=$0.1U^2$<<$UR$. Thus, as a first approximation, the extrema of $c(0,t)$ can be calculated as the extrema of the trigonometric function $f(t)=a_1\cos\left(\frac{U}{R}t\right)-b_1\sin\left(\frac{U}{R}t\right)$ modulated by an exponential decay in time with $R^2/D_{eff}$ for the decay constant. More specifically, if $t_m$ is the time at which $c(0,t)$ is an extremum, $c(0,t_m)= m = a_0 \pm d e^{-D_{eff}t_m/R^2}$ where $d$ is a constant. This demonstrates that in the rotary mixer, $D_{eff}$ can be measured for each experiment as the exponential decay constant of the extrema towards the asymptote value.

In addition, we considered photobleaching, due to the exposure of the fluorescein to light, while analyzing experimental data. We assumed that bleaching led to a linear decay of intensity versus time. Peak values could then be predicted using $c_{bl}(t_m) = a_0 \pm d e^{-D_{eff} t_m / R^2} - K(t_m - t_i)$ where $K$ is the decay constant due to bleaching and $t_i$ is the time at which we started exposing the sample to light (t=0 when mixing starts). Let $t'_n = \frac{t_n + t_{n+1}}{2}$ be the mean time of two consecutive extremum and $c_{bl}(t'_n)$ the mean value of these two peaks, we have $c_{bl}'(t'_n) = \frac{c_{bl}(t_n) + c_{bl}(t_{n+1})}{2} \approx a_0 - K(t'_n - t_i)$ . $K$ was thus recovered experimentally by linear regression on the plot of $c_{bl}(t'_n)$ versus $t'_n$. For each experiment, we then plotted $\ln(abs(\frac{c_{bl}(t_m)}{c'_{bl}(t_m)} - 1)) \approx -\frac{D_{eff} t_m}{R^2}$. $D_{eff}$ was measured using a linear fit of the experimental data.

The mixing time $T_M$ is defined as $\frac{abs(c(\theta, T_M) - a_0)}{a_0} \le M$ .

Let's approximate $c(\theta,t)$ by the first term in the series. Then

$$c(\theta,t) = a_0 + \left[ a_1 \cos\left(\theta - \frac{U}{R} t\right) + b_1 \sin\left(\theta - \frac{U}{R} t\right)\right] e^{-D_{eff} t / R^2}$$

For a given time, the extrema of this function are obtained for $\frac{\partial c(\theta,t)}{\partial \theta} = 0$ . Let's $\theta_M$ and $\theta_M+\pi$ be the solutions of $\frac{\partial c(\theta,0)}{\partial \theta} = 0$. Then $abs(c(\theta,t) - a_0) \le abs[a_1 \cos(\theta_M) + b_1 \sin(\theta_M)] e^{-D_{eff} t / R^2}$ and we obtain $T_M$ as $\frac{abs[a_1 \cos(\theta_M) + b_1 \sin(\theta_M)] e^{-D_{eff} T_M / R^2}}{a_0} = M$ . This leads to $T_M = \frac{R^2}{D_{eff}} \ln(\frac{\alpha}{M})$ with $\alpha = \frac{abs[a_1 \cos(\theta_M) + b_1 \sin(\theta_M)]}{a_0}$.