Gradient Generation

The equations used to predict the replenishment concentration C_s are defined below.

$$C_s = C_0 e^{-t/\tau} \tag{1}$$

τ is defined as the source time parameter:

$$\tau = (V_s\ h_{gel})/(D_{avg}\ A_c) \tag{2}$$

V_s is the volume of solution in the source, h_{gel} is the height of the gel in the port, D_{avg} is average diffusivity of the factor in solution and in the gel, and A_c is the limiting cross-sectional area at the channel entrance.

Time Parameter Definition

Equation 1 is equivalent in form to the discharge equation of a capacitor C through a resistor R. In the case of an RC circuit, the time constant is defined as τ = RC. Here, we define the time constant to be a function of system parameters V_s, h, D_{avg}, and A_c. The following relations must hold true to effectively model the physics of the system:

1. The time constant must increase (longer time to reach $0.63C_0$) when the input volume is increased (increased solute volume), or when the gel height is increased (increased diffusion distance).

2. The time constant must decrease with increasing cross-section area (increased solute flux) at the channel mouth or with increasing diffusivity.

The relation between the parameters is therefore defined by Equation 2. Similar logic was used to define the system time parameter λ (Equation 4).

Example

Consider: $V_s = 3\ \mu$L , $h_{gel} = 0.6$ mm, $D_{avg} = 3.5\ 10^{-4}$ mm^2 sec^{-1}, $A_c = 0.02$ mm^2, then $\tau = 71.4$ hrs. Replenishing the source concentration after 24 hours corresponds to $t = 0.336\tau$ (see Table 1). The replacement concentration is determined by applying Equation 1. The source reservoir should be replenished at this concentration in subsequent replacements. Table 1 is used for reference.

The parameter V_s can be changed during the course of the experiment if desired. The diffusivity can be changed to some degree by changing the system temperature but is generally not practical when biological systems are used.

Gradient Maintenance

The concentration in the source must be replenished periodically to maintain the developed gradient for extended periods of time. The frequency of the source solution replacement ($C = C_s$) is dictated by the system time parameter λ.

Table 1: Determination of replenishment concentration

$C_s = C_0 e^{-t/\tau}$	
$\tau = V_s\, h_{gel} / D_{avg}\, A_c$	
Cs/C_0	t
0.819	0.25τ
0.607	0.5τ
0.368	τ

$$C = C_s e^{-t/\lambda} \tag{3}$$

$$\lambda = (V_s\ L_t)/(D_{gel}\ A_c) \tag{4}$$

$L_t = L + h_{gel}$ corresponds to the total diffusion distance from the source solution to the sink; D_{gel} corresponds to the diffusivity of the species within the gel.

Example

Consider: if $V_s = 3$ μL , $L_t = 1.5$ mm, $D_{gel} = 2\ 10^{-4}$ mm^2 sec^{-1}, $A_c = 0.02$ mm^2, then $\lambda = 312.5$ hrs. Replenishing the source concentration every 24 hours corresponds to $t = 0.0768\lambda$. Equation 3 is used to determine the amount of change in the reservoir concentration. Table 2 is used for reference.

Table 2: Source Depletion as a Function of Replacement Time

$C = C_s e^{-t/\lambda}$	
$\lambda = (V_s\ L_t)/(D_{gel}\ A_c)$	
C/C_s	t
0.99	0.01λ
0.951	0.05λ
0.904	0.1λ

Gradient Profile

The concentration profile that develops is a function of the channel geometry that is used. In the Equation 5, A(x) is the spatially varying cross-sectional area of the channel and x is the spatial coordinate.

$$\frac{1}{A(x)}\frac{d}{dx}\left(A(x)\frac{dC}{dx}\right) = 0 \tag{5}$$

Temporal Dose

The evolution of a transient dose introduced to the system (where the concentration is initially confined to region -h<x<+h) can be predicted with Equation 6.

$$C(x,t) = \frac{C_0}{2}\left\{\operatorname{erf}\left(\frac{(h-x)}{2\sqrt{Dt}}\right) + \operatorname{erf}\left(\frac{(h+x)}{2\sqrt{Dt}}\right)\right\} \tag{6}$$