

SUPPLEMENTAL

A: Here we describe the details of the derivation of the capillary pressure obtained in Eq. 2. The capillary pressure that drives the liquid through a lattice of posts can be obtained by considering the difference in surface energies of the total system before and after it is wetted. Wetting may proceed only if dE is negative.

$$dE = dE_{\text{wetted}} - dE_{\text{non-wetted}} \quad \dots(\text{A1})$$

The following table summarizes the specific areas and the surface energy associated with them before and after wetting. A_{bottom} is the bottom surface where there are no pillars, A_{sidewall} is the surface area of the pillars, and A_{top} is the top surface of the pillars.

Area	Before (non-wetted)	After (wetted)
A_{bottom}	γ_{SV}	γ_{SL}
A_{sidewall}	γ_{SV}	γ_{SL}
A_{top}	-	γ

Table A1

Using Table A1, Eq. (A1) may be expanded to the following.

$$dE = (\gamma_{SL} - \gamma_{SV})A_{\text{bottom}} + (\gamma_{SL} - \gamma_{SV})A_{\text{sidewall}} + \gamma A_{\text{top}} \quad \dots(\text{A2})$$

Assuming unit width in the direction perpendicular into the figure, the area within a length dx would be given as follows. Note that dx will contain several posts.

$$A_{\text{top}} = \frac{\pi d^2}{4 p^2} \cdot dx \cdot 1$$

$$A_{\text{sidewall}} = \frac{\pi d h}{4 p^2} \cdot dx \cdot 1$$

$$A_{\text{bottom}} = \left(1 - \frac{\pi d^2}{4 p^2}\right) \cdot dx \cdot 1$$

...(A3)

Substituting from equation (A3) and noting that $\cos\theta = \frac{\gamma_{SV} - \gamma_{SL}}{\gamma}$, equation (A2)

reduces to the following.

$$dE = -\frac{\gamma}{p^2} \left[\cos \theta \left(\pi dh + p^2 - \frac{\pi d^2}{4} \right) - \left(p^2 - \frac{\pi d^2}{4} \right) \right] \cdot dx \quad \dots$$

(A4)

The surface force is the negative derivation of the surface energy.

$$F = -\frac{dE}{dx} = \frac{\gamma}{p^2} \left[\cos \theta \left(\pi dh + p^2 - \frac{\pi d^2}{4} \right) - \left(p^2 - \frac{\pi d^2}{4} \right) \right]$$

...(A5)

Since pressure is force over the cross-section area, we finally obtain the capillary pressure driving the liquid front as shown below.

$$\Delta P_{cap} = \frac{F}{h \cdot 1} = \frac{\gamma}{p^2 h} \left[\cos \theta \left(\pi dh + p^2 - \frac{\pi d^2}{4} \right) - \left(p^2 - \frac{\pi d^2}{4} \right) \right]$$

...(A6)

B: Effective Pitch: Since the distance between the pillars keeps changing — it is g at the narrowest and p at the widest separation— we define an effective pitch to compare the experimental velocities with what the scaling model predicts

$$p_{eff} = \frac{\text{Area enclosed by four posts}}{\text{pitch}} = \frac{p^2 - \frac{\pi d^2}{4}}{p} = p - \frac{\pi d^2}{4p}$$

C The definition of K_{eff} (now \hat{q}) is non-unique and was chosen for convenience to scale the data obtained from simulations. Other definitions of K_{eff} will give the same scaling result as long as appropriate dimensionless numbers chosen in the model. In our case ΔP is a parameter that we use to specify the pressure at the inlet and outlet of our computation flow domain. We therefore chose $\left(\frac{dP}{dx} \right) \cdot p$ since it equals ΔP . Also, we non-dimensionalized all length scales with the diameter, d , so we then chose d^2 in the expression of K_{eff} .

D: Weak form finite element formulation of the Navier Stokes Equation: To obtain a weak variational formulation, each partial differential equation, i.e x, y, z components of the Navier-Stokes are multiplied with a test function and

integrated over the entire cross-sectional area. The weak formulation obtained from Comsol are as follows.

$$\begin{aligned} & \frac{1}{d^2} \left(\left(-2\mu \frac{\partial u}{\partial x} + p \right) \cdot \text{test} \left(\frac{\partial u}{\partial x} \right) - \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \cdot \text{test} \left(\frac{\partial u}{\partial y} \right) - \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \cdot \text{test} \left(\frac{\partial u}{\partial z} \right) \right) \\ & \frac{1}{d^2} \left(-\mu \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \cdot \text{test} \left(\frac{\partial v}{\partial x} \right) + \left(\left(-2\mu \frac{\partial v}{\partial y} + p \right) \right) \cdot \text{test} \left(\frac{\partial v}{\partial y} \right) - \mu \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \cdot \text{test} \left(\frac{\partial v}{\partial z} \right) \right) \\ & \frac{1}{d^2} \left(-\mu \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right) \cdot \text{test} \left(\frac{\partial w}{\partial x} \right) - \mu \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) \cdot \text{test} \left(\frac{\partial w}{\partial y} \right) + \left(-2\mu \frac{\partial w}{\partial z} + p \right) \cdot \text{test} \left(\frac{\partial w}{\partial z} \right) \right) \end{aligned}$$

Pressure boundary condition was imposed downstream and upstream from the post, as shown by solid lines. These boundaries were also made periodic. The condition of symmetry ($\partial u / \partial n = 0$) is imposed on the other two boundaries as shown using dashed lines. The post is bounded on the bottom surface where no-slip condition is imposed. The condition of symmetry ($\partial u / \partial n = 0$) is imposed on the top.

Deformed Mesh Application Mode: The Moving Mesh (ALE) application mode allows us to create a model where the mesh corresponding to the geometry changes shape according to prescribed mesh displacement. Hence we only draw the geometry once and the prescribed mesh displacement accounts for different geometrical shapes. In our case, we drew a geometry with a unit diameter while h and p were twice the diameter. Since the Navier-Stokes equations were scaled with respect to d , the mesh displacement dx , dy and dz were prescribed as follows.

$$\begin{aligned} dx &= 2 \left(\frac{p}{2d} - 1 \right) (X + 0.5), \text{ if } X \leq -0.5; \quad dx = 2 \left(\frac{sx}{2} - 1 \right) (X - 0.5), \text{ if } X > 0.5 \\ dy &= 2 \left(\frac{p}{2d} - 1 \right) (Y + 0.5), \text{ if } Y \leq -0.5 \\ dz &= \left(\frac{h}{2d} - 1 \right) Z \end{aligned}$$