## Supplementary data

In order to predict the time required to pump a droplet through a highly resistive channel, we begin with the following definitions and approximations.

Equation 1 describes the volume of a droplet on a surface given the radius of the droplet and the contact angle of the fluid with the surface ( $\theta = 90^\circ \rightarrow$  hemisphere,  $\theta = 180^\circ \rightarrow$  sphere).

$$V(\theta) = \pi r^{s} \left[ \left( \frac{1}{3} \cos^{s} \theta - \cos \theta \right) + \frac{2}{3} \right]$$
(1)

Water and PBS have a contact angle of 110° on PDMS. Given this piece of information, we can see that the volume and radius of such a drop can be approximated by (2).

Equation (2) can then be used to obtain (3) which estimates the radius of a droplet of water or PBS on PDMS given the droplet volume.

$$r_{\mu\nu\mu\nu} \cong \sqrt[4]{0.32 \cdot V_a}$$
 (3)

We are able to then estimate the internal pressure of a droplet using the Young-Laplace equation (4)

$$\Delta P_{\rm w} = \frac{2\gamma}{\Gamma_{\rm FDMS}} \quad (4)$$

where  $\gamma$  is the surface tension of the fluid ( 0.072 N/m for water or PBS at 25° C). By putting equation (3) into (4), we can calculate a pressure of our droplet on the PDMS surface. This pressure represents the pressure that drives the passive pumping flow. The flow rate, Q, within the channel is proportional to the driving pressure and is related by (5) where R is the resistance of the channel.

$$\Delta \mathbf{P} = \mathbf{R} \cdot \mathbf{Q} \quad (5)$$

As our goal is to eventually predict the time required for a droplet to completely pump, we are interested in the average pressure during passive pumping. We have chosen to approximate the average pressure as being half of the initial driving pressure ( $P_a \approx P_i/2$ ) where the initial driving pressure is calculated as the difference in pressure between the inlet and outlet drops. We approximate the resistance in the microchannel using (6) where  $\mu$  is the fluid viscosity and L, w, and h represent the length, width, and height of the channel, respectively.

$$\mathbf{R} = \frac{\mathbf{12}\mu\mathbf{L}}{\mathbf{wh^3}} \quad (6)$$

Given that the time, T, to pump a volume V at a flow rate of Q is V/Q and  $Q = \mathbf{R}$ , we can write (7) which describes the pumping time for a timer channel.

ΔP

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$$T_{\rm P} \simeq \frac{V_{\rm in}}{Q} = \frac{V_{\rm in} \cdot R}{\Delta P_{\rm in} - \Delta P_{\rm out}} = \frac{R \cdot V_{\rm in}}{2\gamma \cdot \left(\frac{1}{\Gamma_{\rm in}} - \frac{1}{\Gamma_{\rm out}}\right)} \times 2 = \frac{R \cdot V_{\rm in} \cdot r_{\rm in} r_{\rm out}}{\gamma \cdot (r_{\rm out} - r_{\rm in})}$$
(7)

## Abbreviations

$T_{\rm p}$	pumping time
$p_{\mathrm{a}}$	average pressure
$p_{ m i}$	initial pressure
$\Delta p_{ m in}$	pressure at input port
$\Delta p_{ m out}$	pressure at output port
$\Delta p_{ m w}$	difference pressure air to water
$\Delta p$	difference pressure input port to output port in microchannel
$V_{110^\circ}$	volume of droplet on the PDMS surface (V <sub>PDMS</sub> )
$V_{in}$	volume of droplet at input port
V <sub>out</sub>	volume of droplet at output port
$V_d$	volume of droplet
r <sub>in</sub>	radius of droplet at input port
r <sub>out</sub>	radius of droplet at output port
r <sub>PDMS</sub>	radius of droplet on the PDMS surface