

Diffusion Profile Derivation:

Mass Balance on the device reservoir:

$$dM_r/Dt = -m,$$

where m is the rate of mass efflux and M_r is the mass in the reservoir.

$$dM_r/dt = -A \cdot J,$$

where A is the area for flux and J is the flux as defined by Fick's law.

$$dM_r/dt = A \cdot D \cdot dC/dx,$$

where C is the concentration in the reservoir and x is the direction normal to the interfacial area. Assume: The change in concentration that drives diffusion occurs over the thickness of the chip.

The concentration profile across the chip is linear (i.e. the time scale to achieve a linear profile is much less than the rate of mass efflux.) The external bath is large enough and replaced frequently enough such that it remains at sink conditions.

$$dM_r/dt = -A \cdot D \cdot M_r/(l \cdot V),$$

where V is the volume of the reservoir and l is the thickness of the chip. Solving the first order ODE and applying the initial condition $M_r(0) = M_0$ we get:

$$M_r/M_0 = \exp(-k \cdot t),$$

$$k = A \cdot D/(l \cdot V),$$

Mass Balance on the environment:

$$dM/dt = -dM_r/dt,$$

where M is the mass released into the environment. Solving the ODE results in:

$$M = C - M_r,$$

where C is a constant of integration. Set the initial condition of $M(0) = 0$:

$$M(t) = 1 - \exp(-k \cdot t),$$

$$k = A \cdot D/(l \cdot V).$$

Supplementary Material (ESI) for Lab on a Chip

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