Supplementary Material: Self-Assembled Magnetic Filter for Highly Efficient Immunomagnetic Separation

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MAGNETOPHORESIS

Magnetophoresis is the movement of a magnetically susceptible object in a nonuniform magnetic field. When a spherical object of radius *a* and magnetic susceptibility χ is placed in a magnetic field $\vec{B}$, the induced magnetic moment of the object is

$$\vec{m} = \frac{4\pi a^3 \chi}{3\mu_o}\vec{B}, \quad \textbf{(S1)}$$

where μ_0 is the vacuum permeability. The object will then experiences a force given by:[1]

$$\vec{F}_{MP} = \frac{\vec{\nabla}(\vec{m}\cdot\vec{B})}{2} = \frac{2\pi a^3 \chi}{3\mu_o}\vec{\nabla}\vec{B}^2. \quad \textbf{(S2)}$$

At sufficiently large fields $\left|\vec{B}\right| > \left|\vec{B}\right|_{SAT}$ the moment of a magnetically susceptible object saturates with a magnetic moment m_{SAT}, and the force becomes

$$\vec{F}_{MP} = m_{SAT}\vec{\nabla}B \quad \textbf{(S3)}$$

For the magnetic beads and particles used in this paper, the magnetic moments saturate at B ~ 0.5 T.[2] The self assembled magnetic filter can create such fields (~0.5 T) very close to its surface, in the range of $d < a/2\pi$, where *a* is the lattice spacing of the self assembled magnets (**Fig, 2a**). We therefore used **Eq. S3** to estimate the magnetic force on the trapped objects right on top of the device. Note that the measured trapping force is still strong (~5 nN for MyOne beads) due the large field gradient by the self-assembled magnets. Away from the surface, the force decays exponentially (~$e^{-2\pi d/a}$); across the fluidic channel, **Eq. S2** is a good approximation to calculate magnetic force.

MAGNETIC MULTIPOLES

Herein, we calculate and analyze the magnetic field created by a two dimensional array of magnetic dipoles. This idealized model can be used to aid in the design and characterization of the self-assembled magnet.

The field from an individual magnetic dipole is given by the expression:

$$\vec{B}_m(\vec{r},\vec{c}) = \frac{\mu_o}{4\pi}\frac{3\vec{r}(\mu\cdot(\vec{r}-\vec{c}))-\mu(\vec{r}-\vec{c})^2}{(\vec{r}-\vec{c})^5} \quad \textbf{(S4)}$$

where *c* is the location of the magnetic moment and *r* is the location at which the field is measured. Consider a periodic array of magnetic dipoles, arranged in a 2d square lattice with a spacing *a*. The total magnetic field can be expressed as the superposition of the field from each magnetic moment.

$$\vec{B}_m(\vec{r}) = \sum \vec{B}_m(\vec{r},\vec{c}_{i,j}) \quad \textbf{(S5)}$$

In **Fig. S1a** we plot the magnetic field strength created by the lattice of magnetic dipoles a distance $d = a$ from the lattice. The field strength forms an egg carton pattern, creating many magnetic traps that have a size and periodicity set by the lattice spacing *a* and the distance *d* from the lattice.

To form an analytical expression for the field at distances close to the array, we take advantage of the periodicity of the array and express the field as a sum of periodic functions in a Fourier series, to find the solution:[3]

$$B_m(r) = \frac{8\pi^2\mu_o}{a^2} e^{-2\pi Q|z|}[A\cos(2\pi kx/a)\cos(2\pi ly/a) + B\cos(2\pi kx/a)\sin(2\pi ly/a) + C\sin(2\pi kx/a)\cos(2\pi ly/a) + D\sin(2\pi kx/a)\sin(2\pi ly/a)] \quad \textbf{(S6)}$$

where the term $k = l = 0$ is excluded, and the terms A,B,C,D are set by the boundary conditions,[1] and

$$Q_{l,k} = \sqrt{\left(\frac{k}{a}\right)^2 + \left(\frac{l}{a}\right)^2} \quad \textbf{(S7)}$$

The magnetic force [~(**B**·□)**B**] is plotted *vs.* x at a height $d = a$ from the lattice in **Fig. S1b**. In this plot it can be seen that when $d = a$ the magnetic force is dominated by the first terms of the Fourier series, having a periodicity of *a.* At distances d > a/2 the (k,l) = (0,1), (1,0) harmonics dominate, creating traps with a periodicity of *a.* At distances *d* close to the lattice, higher frequency harmonics become comparable in magnitude to the (0,1) and (1,0) harmonics, leading to magnetic traps with a size and periodicity less than *a*. In the limit $d \rightarrow 0$, the Fourier series that describes the magnetic force approaches a 2d lattice of delta functions with a periodicity of *a.*

Figure Captions

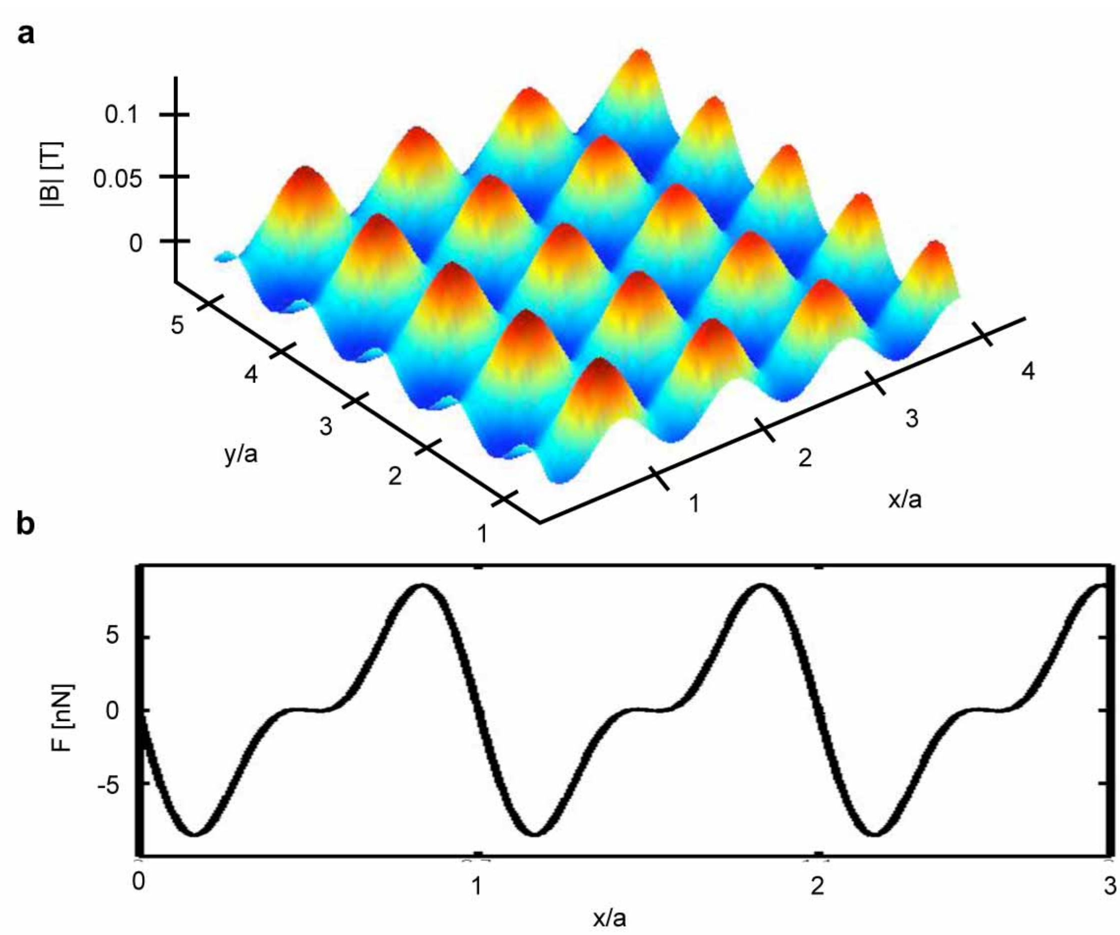

Figure S1: A two dimensional square lattice of anti-aligned magnetic dipoles, as was used in Fig. 1a. (a) A 2D slice of the magnetic field B, a distance *d* = *a* = *8 μm* from the lattice. The magnetic field B creates an egg carton shaped pattern above the array with a periodicity of *a*. (b) A 1D slice of the trapping force in plane with the magnet's surface **is plotted vs*. x* at a height *d* = *a* from the lattice. The magnetic force can be seen to be dominated by a periodicity of *a,* but higher harmonics are clearly visible.**

1. H. Lee, A. M. Purdon, V. Chu, and R. M. Westervelt, Controlled assembly of magnetic nanoparticles from magnetostatic bacteria using microelectromagnet arrays. Nano Lett. **4,** 2004, 995-998.
2. G. Fonnum, C. Johansson, A. Molteberg, S. Morup, E. Aksnes, Characterisation of Dynabeads by magnetization measurements and Mossbauer spectroscopy. *Journal of Magnetism and Magnetic Materials*, 2005, **293**, 41-47.
3. E. V. Rozenfel'd, *Physics of the Solid State*, 2000, **42**, 1680-1687.