Electronic Supporting Information

Microfluidic Operations and Networks Using Knotted Yarns

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Calculation of the fluid concentration in the outlets of the serial dilutor

With reference to Fig. 5C, we perform a nodal analysis to calculate the flow rate ratio and concentration C_4 at outlet 4 (and the complementary concentration C_7 at outlet 7) as a function of the ratio between the resistance of one branch *r* and the outlet resistance given by *nr* with *n* being a proportionality factor. First we write the equations for the current at node *A* and *B* based on the unknown potential P_A and P_B:

$$\begin{cases} \frac{P_1-P_A}{(n+1)r}+\frac{P_1-P_A}{nr}+\frac{P_B-P_A}{r}-\frac{P_A}{2r}=0 & \text{(S1)} \\ \frac{P_1-P_B}{nr}+\frac{P_A-P_B}{r}-\frac{P_B}{r}=0 & \text{(S2)} \end{cases}$$

The equations can be simplified rewritten as:

$$\begin{cases} P_A\frac{3n^2+7n+2}{2n^2+2n}-P_B=P_1\frac{2n+1}{n^2+n} & \text{(S3)} \\ -P_A+P_B\frac{2n+1}{n}=P_1\frac{1}{n} & \text{(S4)} \end{cases}$$

Multiplying equation S4 by $\frac{n}{2n+1}$ and combining equations S3 and S4 together we obtain:

$$\begin{cases} P_A=\frac{10n^2+10n+2}{4n^3+15n^2+11n+2}P_1 & \text{(S5)} \\ P_B=\frac{14n^3+25n^2+13n+2}{4n^3+15n^2+11n+2} & \text{(S6)} \end{cases}$$

To identify the concentration of the liquid at the exit 4, and 7, we need to determine the ratio of the flow rates of $k=\frac{Q_2}{Q_1}$, where $Q_1=\frac{P_A}{2r}$, and $Q_2=\frac{P_A-P_B}{r}$.

$$\boldsymbol{k}=\frac{Q_2}{Q_1}=\frac{3n^2+n}{5n^2+5n+1} \qquad \text{(S7)}$$

Having the flow ratios, the concentration of fluid 2 in exit 4, C_4, can be approximated using a weighted average of the concentrations of each branch,

$$\boldsymbol{C_4} = \frac{C_1 Q_1 + C_2 Q_2}{Q_1 + Q_2} \qquad \text{(S8)}$$

where $C_1 = 0$.and $C_2 = 0.5$

Substituting the concentrations of the liquids and the flow rates in to the eq. S8 we have

$$\boldsymbol{C_4} = \frac{0.5}{\mathrm{k}+1} \qquad \text{(S9)}$$

and similarly the concentration of fluid at exit 7 is given by the ratios of the mirror flow rates Q_1' and Q_2', and the concentrations C_1' and C_2'. Using the fact that $C_4 + C_7 = 1$ we find:

$$\boldsymbol{C_7} = \frac{C_{1'} Q_{1'} + C_{2'} Q_{2'}}{Q_{1'} + Q_{2'}} = \frac{\mathrm{k}+0.5}{\mathrm{k}+1} \qquad \text{(S10)}$$

Fig. S4 shows how the concentrations of the liquid in exits 4 and 7 vary with respect to n.