Appendix

In this supplementary document to the article, detailed information about our the derivation of our equations of motion, methods for obtaining analytical results in the adiabatic limit, and a brief description of the movies are provided.

A1. Theory

As shown in Figure 1 in the article, we consider a suspension of superparamagnetic beads exposed to a square array of ferromagnetic disks (with lattice period d) that are identical both in size and magnetization. In order to simplify the magnetic field calculation, we treat each disk as a pair of opposite magnetic point poles separated by the disk diameter, i.e., d_M. The magnetic pole distribution of the micro-magnet array can be expressed by Eq. I, where λ_0 is the effective magnetic pole density. The Fourier expansion of Eq. I will yield Eq. 1 in the article.[0]

$$\lambda(x,y) = -\lambda_0 \sum_{m=-\infty}^{+\infty} \sum_{n=-\infty}^{+\infty} \left[\delta(x-nd) - \delta\left(x-\left(nd+d_M\right)\right)\right] \delta(y-md) \qquad \text{(I)}$$

The magnetic scalar potential is governed by Laplacian equation and can be solved by separation of variables.[0] Taking the negative gradient of the scalar potential and including the oscillating field, $\vec{H}(t) = H_0\left[\hat{x}\sin(\omega_x t) + \hat{z}\sin(\omega_z t + \varphi_0)\right]$, the expression of the total magnetic field as Eq. 2 is obtained.

The dimensionless variables $\vec{\xi} = 2\pi\vec{x}/d = \left[\xi_x, \xi_y, \xi_z\right]$ and $\xi_M = 2\pi d_M / d$ are adopted along with the short-hand notations: $N=(n^2+m^2)^{1/2}$, $u_n(\xi_x)=\cos(n\xi_x)-\cos(n\xi_x-n\xi_M)$ and $v_n(\xi_x)=\sin(n\xi_x)-\sin(n\xi_x-n\xi_M)$. The magnetic force on the bead is approximated as the force on a point dipole $\vec{m}$ in a magnetic field gradient, $\vec{F}_M = \mu_0(\vec{m}\cdot\nabla)\vec{H}_{tot}$, such that the magnetic force in 3-dimensional is obtained as Eq. II.

$$F = F_0 \begin{Bmatrix} \displaystyle\sum_{m=-\infty}^{+\infty}\sum_{n=1}^{+\infty}\left[\frac{n}{N}\sin(\omega_x t)u_n(\xi_x) - \sin(\omega_z t+\varphi_0)v_n(\xi_x)\right] n\cdot\cos(m\xi_y)e^{-N\xi_z} \\ \displaystyle -\sum_{m=-\infty}^{+\infty}\sum_{n=1}^{+\infty}\left[\frac{n}{N}\sin(\omega_x t)v_n(\xi_x) + \sin(\omega_z t+\varphi_0)u_n(\xi_x)\right] m\cdot\sin(m\xi_y)e^{-N\xi_z} \\ \displaystyle -\sum_{m=-\infty}^{+\infty}\sum_{n=1}^{+\infty}\left[\frac{n}{N}\sin(\omega_x t)v_n(\xi_x) + \sin(\omega_z t+\varphi_0)u_n(\xi_x)\right] N\cdot\cos(m\xi_y)e^{-N\xi_z} \end{Bmatrix} \qquad \text{(II)}$$

with the forcing magnitude $F_0 = 2\pi\mu_0\bar{\chi}VH_0\lambda_0/d$ and using the magnetic susceptibility of the bead as $\bar{\chi} = 3\cdot\chi/(3+\chi)$ which is consistent with a spherical, linearly magnetizable bead.[0] The x component of the above equation yields Eq. 3.

A2. Asymptotic Analysis

The equation of motion for the bead is obtained from the instantaneous balance between magnetic force and fluid drag. We write this in dimensionless form as:

$$\dot{\xi}_x = \omega_0 \sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty}\left[\frac{n}{N}u_n(\xi_x)\sin(\omega_x t) - v_n(\xi_x)\sin(\omega_z t+\varphi_0)\right] \qquad \text{(III)}$$

where $\dot{\xi}_x = 2\pi\dot{x}/d$ and $\omega_0 = 16\pi^2\mu_0\bar{\chi}a^2\lambda_0 H_0/9\eta d^2$. As shown in the article, the adiabatic solution can be derived by considering the limit of extremely low driving frequencies, in which we can assume the bead's velocity approaches $\dot{\xi}_x = 0$. This approach allows us to derive a direct analytical relationship for the bead as a function of time, which is given as:

$$\frac{\sin(\omega_x t)}{\sin(\omega_z t + \phi_0)} = \frac{\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty} v_n(\xi_x)}{\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty} \frac{n}{N} u_n(\xi_x)} \tag{IV}$$

When we consider the simplest possible periodically charged substrate with monochromatic, unidirectional periodicity (n=1 and m=0), and using phase φ_0=π/2 and d_M=d/2, we arrive at expression (7) in the article.

A3. Brief Description of Supplementary Movies

Eleven video clips are submitted with the article to demonstrate the numerical and experimental results.

SI-1 provides an animation of the time modulated potential energy landscape superimposed on the calculated position of the superparamagnetic for the simulated conditions of ω_x= π rad/s and R_f=7/5, corresponding to Figure 1b in the manuscript.

SI-2 shows the open trajectories of the bead when R_f is 1/1. (Figure 3[A] in the manuscript)

SI-3 shows the open trajectories of the bead when R_f is 3/1. (Figure 3[B] in the manuscript)

SI-4 shows the open trajectories of the bead when R_f is 7/5. (Figure 3[E] in the manuscript)

SI-5 shows the open trajectories of the bead when R_f is 9/5. (Figure 3[F] in the manuscript)

SI-6 shows the open trajectories of the bead when R_f is 5/3. (Figure 3[C] in the manuscript)

SI-7 shows the open trajectories of the bead when R_f is 7/3. (Figure 3[D] in the manuscript)

SI-8 shows the closed trajectories of the bead when R_f is 49/50. (Figure 3[J] in the manuscript)

SI-9 shows the closed trajectories of the bead when R_f is 2/1. (Figure 3[I] in the manuscript)

SI-10 demonstrates multiplexed motion in which the small beads move but the big beads to not, which occurs at a phase of φ_0=150°.

SI-11 demonstrates multiplexed motion in which the big beads move but the small beads to not, which occurs at a phase of φ_0=158°.

References

1. A. Cordoba. Dirac Comb. *Lett Math Phys*. **17**, 191-196 (1989).
2. Jackson, J. D. Classical Electrodynamics (Third Edition). *Wiley* (1998).