Comparison of monodisperse droplet generation in flow-focusing devices with hydrophilic and hydrophobic surfaces

Supplemental Figures

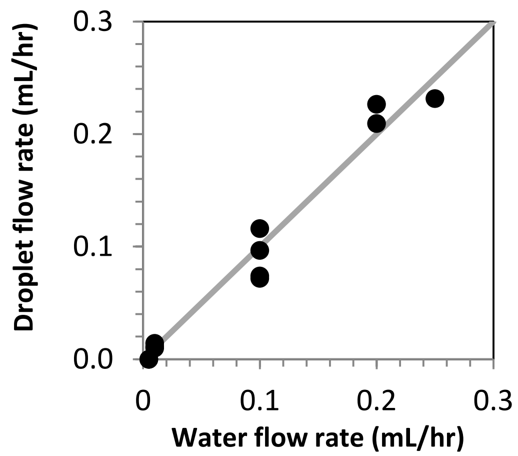

Figure S1. Droplet volumetric flow rate as estimated from image analysis versus the droplet flow rate specified by the syringe pump for case I, water in dodecane.

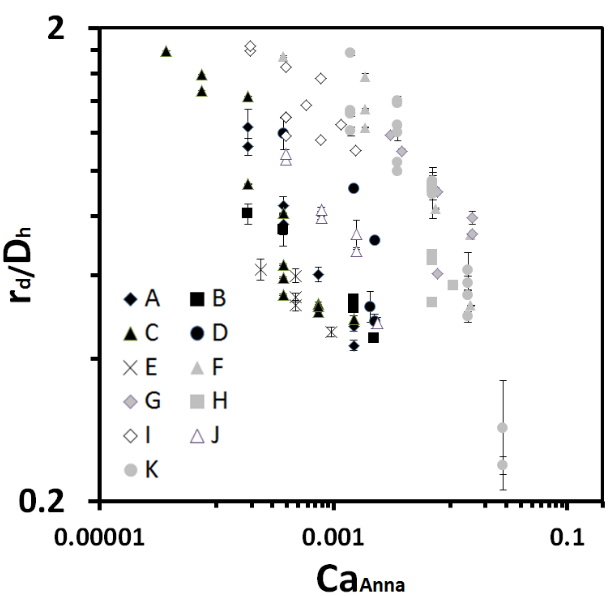

Figure S2. Droplet radius scaled by the orifice hydraulic diameter as a function of fluid capillary number Ca_{Anna}.

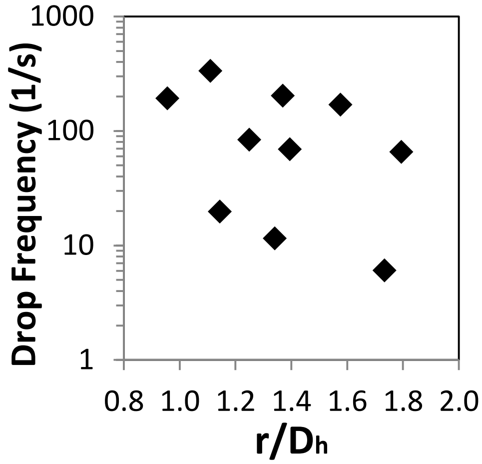

Figure S3. Drop production frequency versus droplet size for case I, water in dodecane.

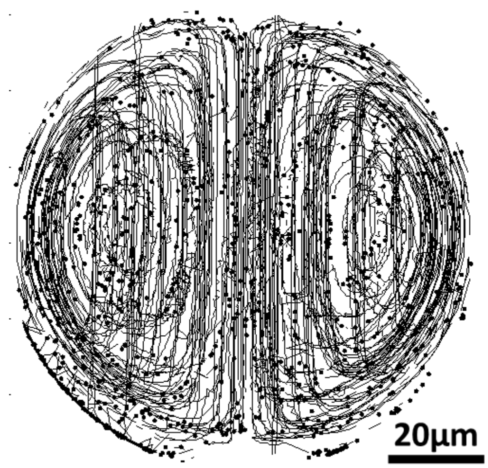

Figure S4. Streamlines within a water droplet containing polystyrene tracer particles. The outer fluid is dodecane, which flows from the top of the image towards the bottom. The coordinate system moves with the drop center. The fluid circulation in this oblate spheroid droplet closely resembles the Hill's vortex that develops inside a spherical droplet moving in a fluid.[1,2] Just inside the inner edge of the droplet, the fluid is being dragged downward by the outer flow. At the bottom of the droplet, the flow is reversed back towards the top of the figure along the centerline.

1. Clift R, Grace JR, Weber ME. *Bubbles, Drops, and Particles*. New York, NY: Academic Press; 1978.

2. Panton RL. *Incompressible Flow*. Hoboken, NJ: John Wiley & Sons; 2005.

Appendix

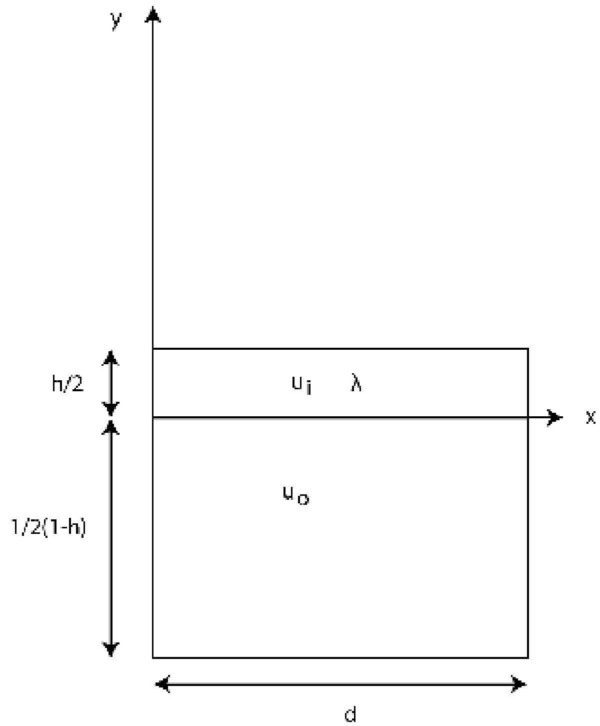

Figure A1. Coordinate system and diagram of inner fluid of velocity u_i surrounded by outer fluid of velocity u_o flowing down a non-dimensional rectangular channel of height d and width 1. This figure is a modified version of that given in Tang and Himmelblau[3].

The curves plotted in Figure A1 were generated from a Fourier-series solution of Stokes equation, modified slightly from the solution given in Tang and Himmelblau[3], for two immiscible fluids flowing down a channel of width 1 and height d. The inner and outer fluid velocities are given by u_i u_o respectively. The viscosity ratio λ is the ratio of the inner fluid to the outer fluid viscosities. Both the inner and outer fluids span the height of the channel in the x-direction, but the inner thread has a width h in the y-direction. The non-dimensional governing equations for this problem are

$$\lambda \nabla^2 u_i = -1$$
$$\nabla^2 u_o = -1, \qquad \text{(A1)}$$

with boundary conditions

$$\frac{\partial u_i}{\partial y} = 0 \; for \; y = \frac{h}{2}$$
$$u_i = u_o \; for \; y = 0$$

$$\lambda \frac{\partial u_i}{\partial y} = \frac{\partial u_o}{\partial y} \; for\; y = 0$$
$$u_i = u_o = 0 \; for\; x = 0, d$$
$$u_o = 0 \; for\; y = -\tfrac{1}{2}(1-h). \qquad \text{(A2)}$$

The problem can be solved through a Fourier series,

$$u_o(x,y) = \frac{2}{d}\sum_{n=0}^{\infty} f_{n,o}(y) Sin\left(\frac{m\,\pi\,x}{d}\right)$$
$$u_i(x,y) = \frac{2}{d}\sum_{n=0}^{\infty} f_{n,i}(y) Sin\left(\frac{m\,\pi\,x}{d}\right) \qquad \text{(A3)}$$

where

$$f_n(y) = \int_0^a u(x,y) Sin\left(\frac{m\,\pi\,x}{d}\right) dx, \qquad \text{(A4)}$$

where $f_{n,o}(y)$ and $f_{n,i}(y)$ are the Fourier series coefficients for the outer and inner fluid respectively. These terms are found by using definition (A4) in differential equations (A1) which yields two ordinary differential equations for $f_{n,o}(y)$ and $f_{n,i}(y)$,

$$\frac{d^2 f_{n,i}}{dy^2} - \left(\frac{m\,\pi}{d}\right)^2 f_{n,i} = -\frac{2d}{m\,\pi\,\lambda}$$
$$\frac{d^2 f_{n,o}}{dy^2} - \left(\frac{m\,\pi}{d}\right)^2 f_{n,o} = -\frac{2d}{m\,\pi}. \qquad \text{(A5)}$$

The solution of equations (A1) – (A2), (A5) are shown in Figure 6. using 100 Fourier terms, integrating the flow over the channel width and depth to obtain volumetric fluxes. No significant differences were seen by using a larger number of terms.

3. Tang, Y.P. and D.M. Himmelblau, *Velocity Distribution for Isothermal Two-Phase Co-Current Laminar Flow in a Horizontal Rectangular Duct.* Chemical Engineering Science, 1963. **18**: p. 143-144.