Electronic Supplementary Material (ESI) for Lab on a Chip

Supplementary material

Highly Enhanced Energy Conversion from the Streaming Current by Polymer Addition

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Derivation of the absolute conversion efficiency in the chip channel

As shown in our previous paper[1], our experimental setup can be divided into three sections including inlet tubing; chip; outlet tubing. An equivalent circuit of this energy conversion system is depicted in Fig. 1S. Each sections can be considered as a constant current source (numbered i) with an internal electrical resistance R_i determined by the cross section (A_i), length (L_i) and solution conductivity ($\gamma = C_i\lambda_i$), (equation 1). The system is finally connected in series with the external resistance R_{ext} which in our case was represented by the voltage source. The resistance of the Ag/AgCl electrodes to charge transport is neglected.

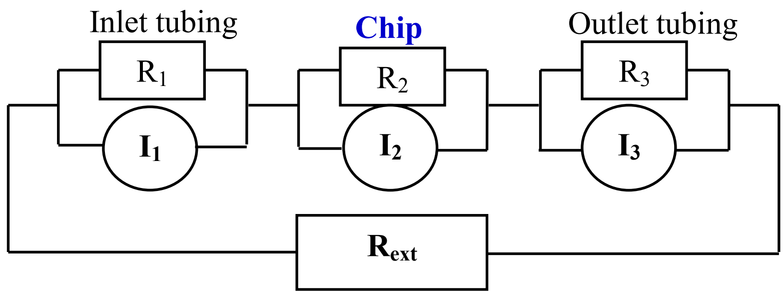

Figure 1S: An equivalent circuit of the energy conversion system

Table 1S: Length and cross sectional area of each section

	L_i	A_i
Inlet tubing	440 mm	2500π $(\mu m)^2$
Outlet tubing	150 mm	2500π $(\mu m)^2$
Microfluidic chip	3.8 mm	400 $(\mu m)^2$

$$R_i = \frac{L_i}{\gamma A_i} \tag{1}$$

C_i is the solution concentration in *i* th section, λ_i is the molar conductivity of *i* th section.[2]

The maximum output power the entire system (connecting capillary/chip/connecting capillary) is ($P_{out\ max}$).[3]

$$P_{out\ max} = \frac{I_s \times \Delta V}{4} \tag{2}$$

According to Kirchhoff's laws, the streaming current of the whole system can be expressed as

$$I_s = \frac{I_1 \times R_1 + I_2 \times R_2 + I_3 \times R_3}{R_1 + R_2 + R_3} \tag{3}$$

At equilibrium, the streaming potential of the entire system is equal to

$$\Delta V = I_1 \times R_1 + I_2 \times R_2 + I_3 \times R_3 \quad (4)$$

Substituting equation (3) and (4) into equation (2), we obtain

$$P_{out\,max} = \frac{(I_1 \times R_1 + I_2 \times R_2 + I_3 \times R_3)^2}{4\,(R_1 + R_2 + R_3)} \quad (5)$$

Substituting values of L_i and A_i from table (1S) into the equation (1), we obtained $R_1 \approx 6R_2$ and $R_3 \approx 2R_2$. On the other hand, we have $I_3 \approx 3I_1$ (streaming current in the same diameter tubing with 1/3 length). Therefore, equation (5) becomes

$$P_{out\,max} = \frac{(I_1 \times 6R_2 + I_2 \times R_2 + I_3 \times 2R_2)^2}{4\,(6R_2 + R_2 + 2R_2)} = \frac{R_2(6I_1 + I_2 + 2I_3)^2}{36} = \frac{R_2(4I_3 + I_2)^2}{36} \quad (6)$$

At this stage, it is interesting to make a comparison between I_2 and I_3. Theoretically, the streaming current for normal electrolyte solution in both capillary tube and rectangular channel depends on the channel dimensions and is proportional to the pressure gradient.[1, 4] The pressure gradient, in turn, depends on the hydraulic resistance (R_{hyd}) of the capillary or the channel in which it is applied.[5] Therefore, the streaming current eventually is proportional to channel dimensions and the hydraulic resistance. Eqs. (7) and (8) show the hydraulic resistance of the capillary (R_{hyd3}) and the rectangular channel (R_{hyd2}) respectively.[5]

$$R_{hyd3} = \frac{8\,\eta L_3}{\pi r^4} \quad (7)$$

$$R_{hyd2} = \frac{12\,\eta L_2}{(1 - 0.63(h/w))} \times \frac{1}{h^3 w} \quad (8)$$

in which, r is the radius of the capillary tubing (μm), L_i is the length of the channel or capillary tubing (mm), h is the channel height (μm), w is the channel width (μm) and η (Pa.s) is the viscosity of the electrolyte solution (KCl 1mM or 0.01 mM).

On substituting values of channel and tubing dimensions from table (1S) into e.q (7) and e.q (8), we obtain that the ratio $R_{hyd2}/R_{hyd3} = 22.14$. Hence, the pressure gradient in the chip channel is much larger than in the capillary. Substituting this result in the equation for streaming current (e.g. equation (1) in ref[1]), we find that the ratio $I_2/I_3 = 44.5$. Thereby, the factor $(4.I_3)$ can be neglected from eq. (6) and the maximal output power $P_{out\ max}$ of the entire system from eq. (6) becomes

$$P_{out\,max} \approx \frac{R_2(I_2)^2}{36} = \frac{1}{9}\, P^{chip}_{out\,max} \qquad (9)$$

In which $P^{chip}_{out\ max}$ is the maximal output power obtained from the microchannel. The maximal absolute energy conversion efficiency we obtained for the entire system is 0.038%. From eq. (9), the maximal energy conversion efficiency of our chip can be derived as 0.34%. Though this still seems low, it is worth stressing that the result from this work was obtained in a 10 micrometer high microchannel where theoretically expected efficiencies would normally be much lower.[6]

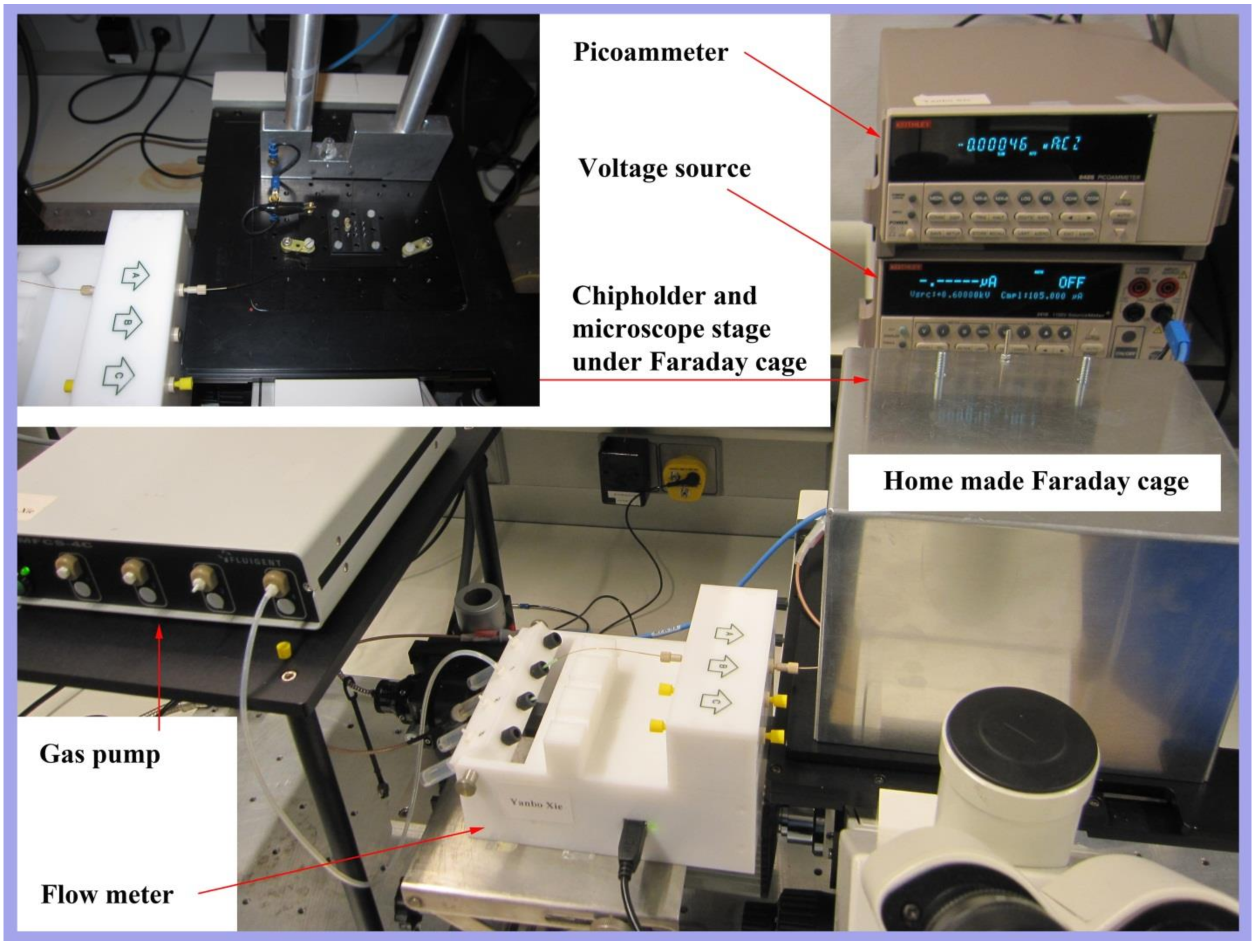

Figure 2S: Experimental setup

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