Electronic Supplementary Material (ESI) for Lab on a Chip

Supplementary Information

Linearity Analysis of Microfluidic Capacitive Membrane

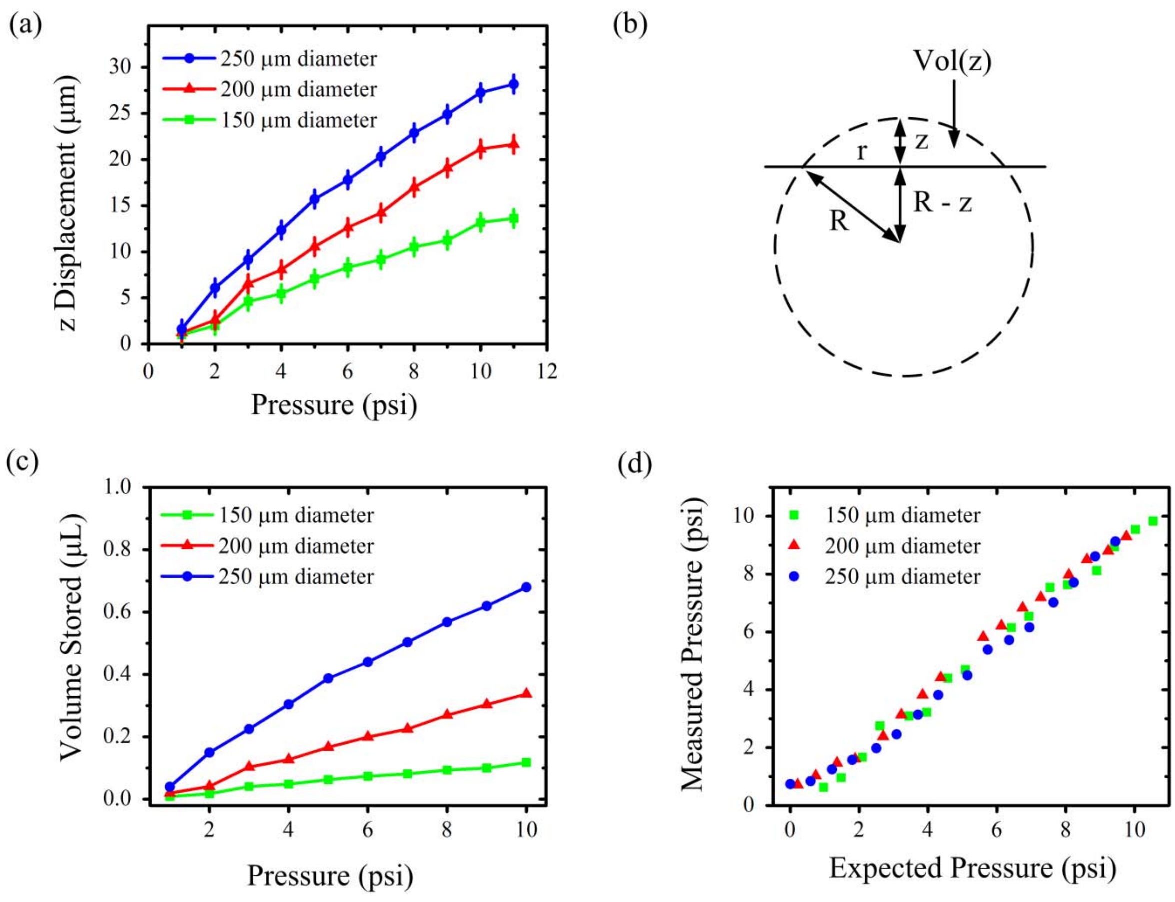
Analogous to electronic capacitors, whose linearity is defined by stored charge as a function of applied voltage, linearity of fluidic capacitors can be investigated by looking at the extra volume of liquid stored for a certain applied pressure Vol(P). This dependence can be further divided into z(P) and Vol(z), where z is the center vertical displacement of the capacitive diaphragm. Supplementary Supplementary Fig. 1a shows the center displacement of fluidic capacitor membranes with respect to applied pressure. Membranes with three different diameters are tested. These diameters are 150 µm, 200 µm, and 250 µm. Pressures in the range of 0 – 10 psi are used in this test because larger pressures resulted in center displacements greater than the available room for membrane deformation (ie. height of the chamber). Results show that vertical displacements as a function of applied pressure are linear for 150 µm and 200 µm diaphragms and less linear for the 250 µm diaphragm. Linear fitting yields R^2 values of 0.98, 0.99, and 0.97 respectively. Because most rounded flow channels in microfluidic devices are smaller than 25 µm tall, this result imply that deformation of the PDMS membrane required to close these flow channels are within the linear regime of the z(P) curves.

$$Vol_{cone} = \frac{1}{3}\pi r^2 (R - z) \quad (1)$$

$$Vol_{solid\ angle} = \frac{4}{3}\pi R^3 \frac{\Omega}{4\pi} \quad (2)$$

$$Vol(z) = Vol_{solid\ angle} - Vol_{cone} = \frac{1}{6}\pi(z^3 + 3r^2 z) \quad (3)$$

Having shown linearity of z(P), we next look at the relationship between additional volume of incompressible fluid stored due to a change in vertical displacement z of the PDMS diaphragm. This is done theoretically using a section of a spherical shell to model the deformation of a PDMS membrane (Supplementary Supplementary Fig. 1b). Additional volume created due to the deformation of a circular PDMS membrane equals the difference between volume of a section of a sphere and volume of a cone that occupies the same solid angle Ω of the sphere. Terms in Supplementary Eqns. 1-3 correspond to Supplementary Supplementary Fig. 1b and shows that additional volume as a function of height z around the point z = 0 is linear when vertical displacement is much smaller than the radius of the diaphragm, which holds for the microfluidic DAC (r > 100 μm whereas z < 30 μm). A worst case ratio of 10:3 in r:z implies that the linear term in Supplementary Eqn. 3 is 10 times greater than the 3rd order term. Therefore, Vol(z) is linear for our displacements of interest. Combining the two relations together, we show that in our relevant region of operation concerning the microfluidic DAC, PDMS membranes with diameters of 150 μm, 200 μm, and 250 μm act as linear fluidic capacitors.



Supplementary Fig. 1 Linearity investigation of the microfluidic DAC output. (a) PDMS diaphragm center vertical displacements z(P) for different diameters (green square: 150 μm, red triangle: 200 μm, blue circle: 250 μm) show linear response to pressure in the region of the DAC operation. (b) Theoretical modeling of the extra volume created due to deformation of the circular PDMS diaphragm. The diaphragm deformation profile is modeled as a section of a spherical shell. The extra volume as a function of vertical displacement Vol(z) is the difference between a section of the sphere and a cone that occupies the same solid angle. Vol(z) is also shown to be linear for $r^2 >> z^2$, which is the region of operation of the DAC (Supplementary Eqn 3). (c) Combining z(P) and Vol(z), PDMS membranes are shown to be linear capacitors. (d) Corrected output transfer function of the microfluidic serial DAC shows uniform transfer functions regardless of size of capacitive diaphragms.

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Iterative Fitting to Quantify Extent of Non-Idealities

In order to quantify the extent of capacitive mismatch and volume injection, we create an iterative fitting method that minimizes RMSE (root mean squared error) of a linear fit by varying capacitor ratios. Transfer function of an ideal serial DAC is a line with slope of 1 that passes through the origin. Mismatched capacitors cause nonlinear output while volume injection results in a vertical offset. Thus, for each membrane diameter, the algorithm sweeps through a range of capacitor ratios C1:C2 and adjust the expected pressure output for each digital pressure input code. Then, a line with slope of 1 is fitted to the adjusted data (actual pressure output verse expected pressure output) to obtain offset and RMSE. The measured C1:C2 ratio is taken to be one that results in the smallest RMSE, and the corresponding offset is the increase in pressure due to volume injection. Finally, capacitor ratios and volume injection caused pressure increases are taken together to adjust expected pressure output in the transfer function (Supplementary Supplementary Fig. 1d). Compared to Fig. 3, the adjusted transfer function shows more consistent DAC output that are not affected by fluidic capacitor sizes. This is also consistent with what we expect after correction for mismatch and volume injection. The best-fit C1:C2 ratios obtained for 150 µm, 200 µm, and 250 µm are 1.19, 1.17, and 1.06. As diaphragm sizes increase, ratios get closer to unity because channel capacitance mismatch takes up smaller portions of the total capacitance. In terms of volume injection, we obtained resulting pressure offsets of 0.55, 0.12, and -0.52 psi (less than half of a LSB) for membrane diameters of 150 µm, 200 µm, and 250 µm respectively. Pressure offsets decrease with increasing diaphragm sizes due to less significant deformations caused by the same injected volume. The negative pressure offset for the 250 µm membrane is most likely due to inherent nonlinearity associated with this fluidic capacitor. Compared to an ideal DAC output, capacitor whose capacitance decreases with increasing pressure result in output with slopes less than 1. Therefore, by fitting a straight line with slope of 1 to the 250 µm DAC output data, we create a situation where the intercept becomes negative. Back calculating the volume displaced during injection using data from 150 µm and 200 µm DACs we obtain ~15 pL, which is a reasonable result.