Electronic Supplementary Information: Flow switching in microfluidic networks using passive features and frequency tuning

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Derivation of governing equations

The motion of fluid in the tube, which dictates the deformation of the film at the tube/chip interface and drives flow, can be described as follows. Force balance for the actuator assembly is given by:

$$\sum F_x = m_v \ddot{\delta}_a = F_i(\omega t) - k_v \delta_a - c_v \dot{\delta}_a - A_i p_1 \tag{S1}$$

where m_v is the effective mass of the voice coil shaft, $\delta_a(t)$ is the displacement of the voice coil shaft, $F_i(\omega t)$ is the electromagnetic force applied to the voice coil shaft, k_v is the stiffness of the voice coil springs, c_v is the effective damping constant for the voice coil (due to eddy currents and friction), and A_i is the area of the input syringe film. The term $A_i p_1$ reflects the back pressure acting on the syringe film generated by fluid in the tube.

Next, consider flow in the syringe cap and entrance of the tube. As shown schematically in Fig. 1C, V_a is the volume associated with motion of the actuator, which gets split into a volume stored in the bulging annulus formed around the actuator piston, V_s, and that injected into the tube, V_i: hence, $V_a = V_s + V_i$. The volume injected into the tube gets divided into that stored due to the capacitance of the tube near the inlet V_1 and that which travels through the tube, V_t, such that: $V_a = V_s + V_1 + V_t$. The capacitance of the annulus (C_s) and tube (C_t) relates these volumes to the pressure in the syringe cap: $V_s = C_s p_1$ and $V_1 = C_t p_1$. Using these relationships, one obtains:

$$V_a = (C_s + C_t)\, p_1 + V_t \tag{S2}$$

The flow through the tube is divided into the fluid stored near the output capacitor and the output capacitor itself, such that $V_t = V_2 + V_o$. Using the relationship for the pressure at the outlet, we have $p_2 = V_o/C_o = V_2/C_t$

where C_o is the capacitance of the output (chip) capacitor. This implies:

$$V_t = \left(1 + \frac{C_t}{C_o}\right) V_o \tag{S3}$$

The above two relationships yield:

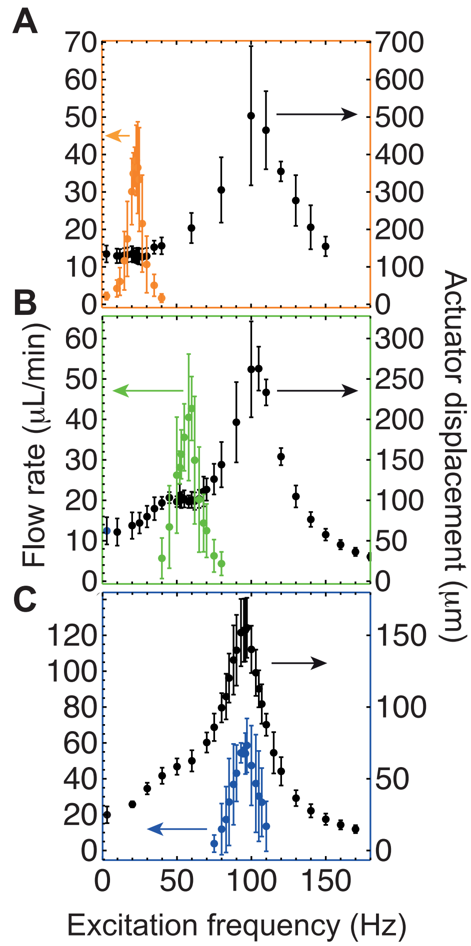
$$V_a = (C_s + C_t)\, p_1 + \left(1 + \frac{C_t}{C_o}\right) V_o \tag{S4}$$

The axial flow in the tube is described by $p_1 - p_2 = L\ddot{V}_t + R\dot{V}_t$, where L is the fluidic inductance associated with the tube, and R is the tube's fluidic resistance. Replacing V_t with the expression above, and noting $p_2 = V_o/C_o$, we have:

$$p_1 = \left(1 + \frac{C_t}{C_o}\right) \left[L\ddot{V}_o + R\dot{V}_o\right] + \frac{V_o}{C_o} \tag{S5}$$

Electronic Supplementary Material (ESI) for Lab on a Chip

Frequency response of actuator and flow rate



Supplementary Figure 1: Comparison of average flow rate and measured input actuator displacement *versus* driving frequency for (A) 400 mm (B) 80 mm and (C) 50 mm input tubes. Error bars represent standard deviation from five measurements.