Electrokinetic Tweezing: Three-Dimensional Manipulation of Microparticles by Real-Time Imaging and Flow Control

Zachary Cummins,*[a] Roland Probst[b] and Benjamin Shapiro[a,c]

Electronic Supplementary Information (ESI)

Simulation of the EP and EO forces on a particle steered along a trajectory

The electric field in the water-filled device was simulated in COMSOL Multiphysics 3.5a (COMSOL, Inc., Burlington, MA, USA) and extracted in MATLAB R2007b (MathWorks, Natick, MA, USA) for use with control program. To determine the relative contributions of EP and EO to the net EK velocity in the device, we compared the measured velocity of the particle with the electric field simulated for the device. Using equation (3) of the main paper,

$$\vec{v}_{EK} = \mu_{EK}\vec{E} \,,$$

with the data shown in **Fig. 5** of the main paper, we estimate a net EK mobility of $\mu_{EK} \approx -17.6 \times 10^{-9}\ \mathrm{m^2V^{-1}s^{-1}}$ for the polystyrene microspheres suspended in DI water in our PDMS microfluidic device. From our previous work[3] we estimate that the EO mobility in the device is $\mu_{EO} \approx 36.5 \times 10^{-9}\ \mathrm{m^2V^{-1}s^{-1}}$, which leads to an EP mobility of $\mu_{EP} \approx -54.1 \times 10^{-9}\ \mathrm{m^2V^{-1}s^{-1}}$ for the microspheres, an estimate that is consistent with measurements made in that work for similar microspheres. Using these mobilities, we show in **Fig. S1** the fields required to move a particle along the vertical plane ∞ trajectory shown in the inset in panel (b) of **Fig. 2**.

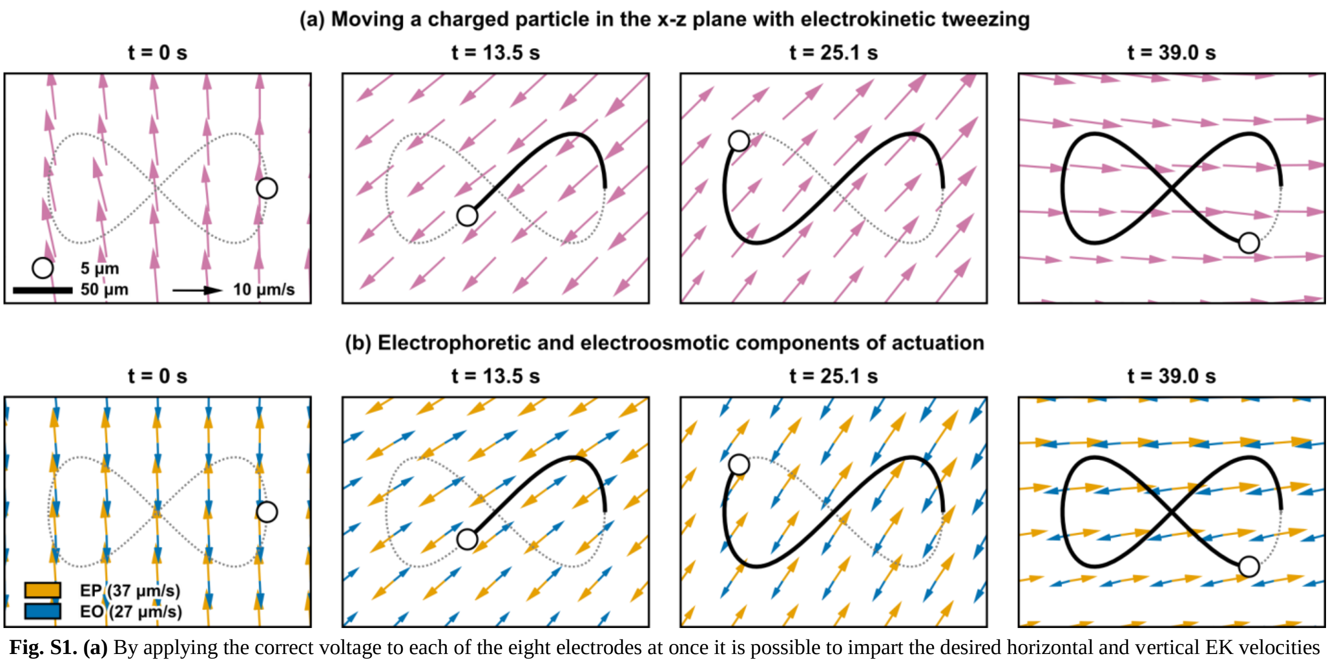

Fig. S1. (a) By applying the correct voltage to each of the eight electrodes at once it is possible to impart the desired horizontal and vertical EK velocities to a particle at any location. For example, here we show the EK velocity fields created while steering a particle along the vertical ∞ trajectory shown in panel (b) of **Fig. 2** of the main text. Note that at each time the EK velocity is pointed along the tangent of the desired trajectory at the location of the particle (black circle). **(b)** For a negatively charged particle, the velocity components due to EP (orange) and EO (blue) oppose each other, but since their sum is usually non-zero the control algorithm can use their combination (the magenta arrows in panel a) to manipulate any single particle as desired.

Kalman filter formulation

A Kalman filter[1] was used to smooth noisey measurement data from the 3D imaging algorithm and predict the position of the particle in the next time step. We assume for simplicity that during each time step that a particle moves in straight line with the velocity described by equation (4) in the main text. Therefore we use the following difference equation to define the dynamics of our system:

$$\begin{bmatrix} \vec{x} \\ \dot{\vec{x}} \end{bmatrix}_{k+1} = \begin{bmatrix} \mathrm{I} & \Delta t \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \vec{x} \\ \dot{\vec{x}} \end{bmatrix}_{k} + \begin{bmatrix} 0 \\ \mathrm{I} \end{bmatrix} \vec{v}_{EK,k+1}$$

where $\vec{x} = \begin{bmatrix} x & y & z \end{bmatrix}^T$ is the position of the particle and Δt is the period of the control loop (0.05 s for a 20 Hz control loop).

The measurement and process noise covariance matrices of the filter, Q and R respectively, were estimated using an auto-regressive filtering technique[2] that yielded

$$Q = \begin{bmatrix} 2.347e-12 & 2.371e-11 & 1.524e-12 & 0 & 0 & 0 \\ 2.371e-11 & 3.921e-10 & 2.099e-11 & 0 & 0 & 0 \\ 1.524e-12 & 2.099e-11 & 1.948e-12 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$R = \begin{bmatrix} 1.939e-13 & 3.398e-14 & 3.759e-13 \\ 3.399e-14 & 2.025e-13 & 8.008e-13 \\ 3.759e-13 & 8.008e-13 & 5.712e-12 \end{bmatrix}.$$

During the experiment shown in **Fig. 5** of the main text, the error covariance matrix P converged to

$$P = \begin{bmatrix} 1.556e-13 & 4.080e-15 & 1.384e-13 & 0 & 0 & 0 \\ 4.080e-15 & 1.245e-13 & 2.480e-13 & 0 & 0 & 0 \\ 1.384e-13 & 2.480e-13 & 1.768e-12 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Notes and References

[a] Fischell Department of Bioengineering, University of Maryland, College Park, MD 20742, United States. E-mail: zcummins@umd.edu
[b] ACUITYNANO, Rockville, MD 20850, United States.
[c] Institute for Systems Research, University of Maryland,College Park, MD 20742, United States.

1. http://www.mathworks.com/matlabcentral/fileexchange/38302-kalman-filter-package
2. R. E. Kalman, *J. Basic Eng.*, 1960, **82**, 35–45.