Supplementary Material: The microfluidic Kelvin water dropper

Relation between the electrical charge in the droplet and the electric field in the system

Figure 5 in the main text of the article shows the stability curve for a charged droplet subjected to an homogeneous electric field as given by Fontelos et al. (Ref [15]). For comparison with that theoretical prediction, in this section we look for a relation between the charge carried by a droplet and the electric field present in our microfluidic system. Figure 1 shows a sketch of the problem domain where we consider the two inductor electrodes, a spherical droplet between the electrodes and the jet. The jet is grounded and the electrical charge induced on its surface is proportional to the potential of the inductor electrode. We assume that this is the charge carried by the droplet after it detaches.

The electric potential in the problem domain is solution of Laplace equation, $\nabla^2\phi = 0$. We make use of the symmetry of the problem and solve for the electric potential in half of the domain. The potential on the inductor electrodes is 1 V and the potential of the droplet is, in principle, unknown. However, if we impose that the charge on the droplet is equal to the charge on the jet surface, the electrostatics problem is determined and we can find the potential of the droplet. With this goal, we write the electric potential in our domain as $\phi = \phi_1 + V_g\phi_2$, where ϕ_1 and ϕ_2 are the solution to the problems 1 and 2 shown in Figure 2 and V_g is the droplet potential. The charge on the jet (Q_{jet}) is then obtained as:

$$Q_{\text{jet}} = \int_{S_{\text{jet}}} \boldsymbol{D}_1 \cdot \mathrm{d}\boldsymbol{S} + V_g \int_{S_{\text{jet}}} \boldsymbol{D}_2 \cdot \mathrm{d}\boldsymbol{S} \tag{1}$$

where $\boldsymbol{D}_1$ and $\boldsymbol{D}_2$ are the electric displacement fields in problems 1 and 2, respectively, and S_{jet} indicates that the integral is performed over the surface of the jet. Similarly, the charge on the droplet (Q_{droplet}) is obtained as:

$$Q_{\text{droplet}} = \int_{S_{\text{droplet}}} \boldsymbol{D}_1 \cdot \mathrm{d}\boldsymbol{S} + V_g \int_{S_{\text{droplet}}} \boldsymbol{D}_2 \cdot \mathrm{d}\boldsymbol{S} \tag{2}$$

S_{droplet} indicates that the integral is performed over the droplet surface. The

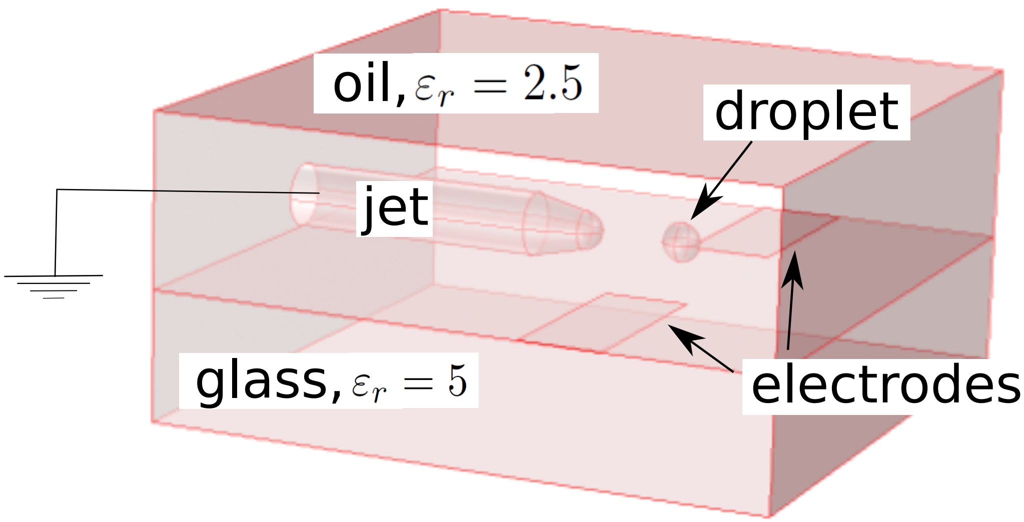

Figure 1: Schematic drawing of the problem domain showing the jet, the inductor electrodes and a droplet. Droplet radius is $45\,\mu m$ and the other dimensions are taken from the experimental device.

droplet potential is found by equating equations 1 and 2.

We have solved the problem using COMSOL, a comercial software implementing the finite element method, and we obtained $V_g = -1.145\,\mathrm{V}$. Figure 3 shows the solution to the electric potential ϕ on two slices of the 3D domain. We obtained that the charge on the droplet is $Q_{\text{droplet}} = -2.54 \times 10^{-14}\,\mathrm{C}$ and the maximum magnitude of the electric field on the droplet surface is $E(max) = 57100\,\mathrm{V/m}$. We acknowledge that the situation here is different of that in Fontelos et al. (Ref. [15]), where an homogeneous electric field is applied, but for the sake of comparison between both situations, we take $E(max)$ as a representative value of the electric field magnitude in our system.

Using nondimensional quantities as mentioned in the main text[1], the following relation is found:

$$E^* = 2.52\sqrt{X} \tag{3}$$

[1] $(X = Q^2/(32\pi^2\varepsilon_{oil})\gamma R^3,\ E^* = E\sqrt{\varepsilon_{oil}R/\gamma})$

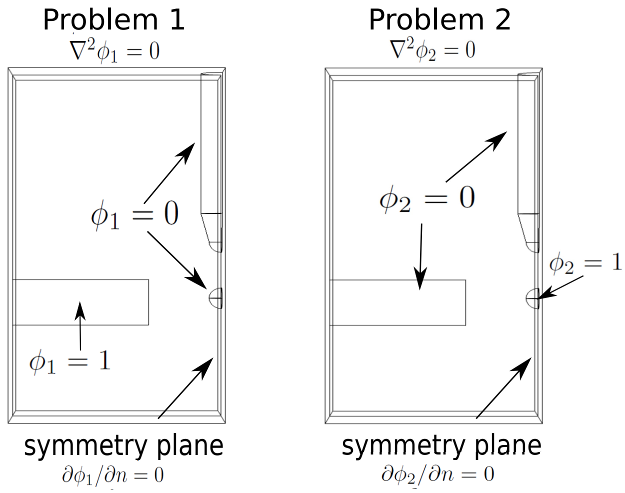

Figure 2: Problem domain indicating the boundary conditions for the electric potential on the jet, droplet and electrode. Laplace equations is solved for the two problems and the total electric potential is obtained as $\phi = \phi_1 + V_g\phi_2$.

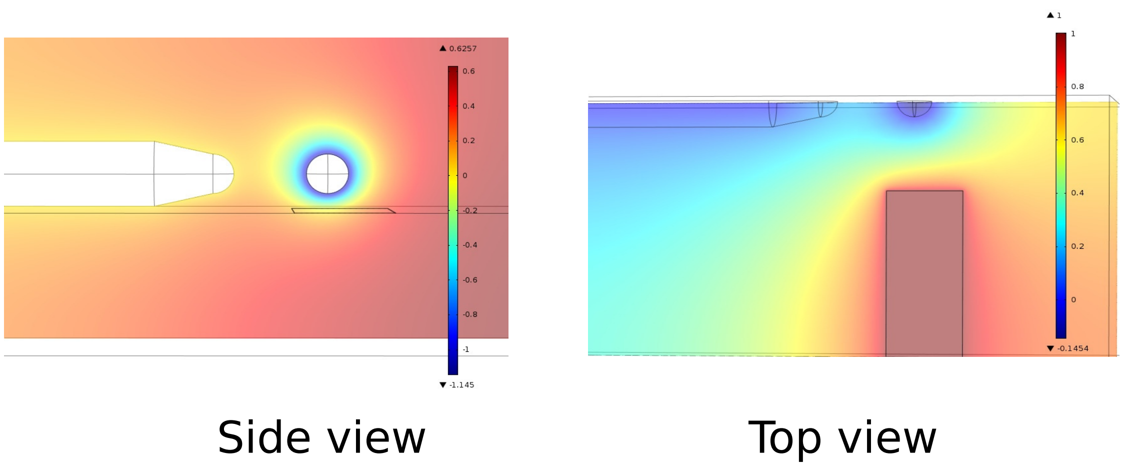

Figure 3: Solution for the electric field potential ϕ in the microfluidic sytem.