

Inertial Microfluidic Physics

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Table S1. Mechanisms of particle lateral migration

Category	Mechanism	General migration direction*
Lift force	Dominant inertial lift	shear gradient lift Channel wall
		Wall effect lift Center
	Weaker inertial lift	slip-shear lift Lagging: center Leading: channel wall
		slip-spin lift Center
	Viscoelastic flow	Elastic lift Center and corners
Deformable particle		Lift due to surface forces Center
		Deformability-induced lift Center
Drag force	Dean flow	Secondary flow due to fluid inertia Near center: outward** Near wall: inward**
	Grooves	Secondary flow due to symmetry breaking Direction of groove slope
Buoyant force	Density mismatch	Inertia of particle $\rho_p > \rho$: outward** $\rho_p < \rho$: inward**

* In the case of typical rectangular microchannel flows.

** Outward/inward: radially outward/inward direction considering curvature of a curving channel.

Table S2. Inertial Microfluidic Foundations

Inertial Microfluidic Foundations	Analytically confirmed	Numerically confirmed	Experimentally confirmed
Inertial lift scales with $\rho U^2 a^4/H^2$ for $a/H \ll 1$	X	X	O
Inertial lift scales with $\rho U^2 a^3/H$ near the channel center for finite-size particle	O	X	O
Inertial lift scales with $\rho U^2 a^6/H^4$ near the channel wall for finite-size particle	O	X	O
Four equilibrium positions in square channels	O	X	X
Two dominant (stable) equilibrium positions in rectangular channels	O	X	X
Slight shift of focusing positions towards the walls at increased Re	O	O	X
Formation of new focusing streams at high length fractions (i.e. $\phi \gtrsim 75\%$) due to particle interactions	O	O	X
Length required for focusing in straight channel scales as $\frac{\pi \mu h^2}{\rho U_m a^2 f_L}$, with the average f_L about 0.02-0.05 for channel aspect ratios (<i>height/width</i>) between 2 and 0.5	X	O	O
Particle migration towards center or the walls in pressure-driven flow in non-Newtonian fluid	O	O	X
Dependence of equilibrium position on rotational diameter and independence of cross-sectional shape for axially symmetric particles	O	O	X
Deformability-induced lift near the channel center scales with $Ca \mu U a^3 d/H^3$	X	O	O
For small and large λ_d (viscosity ratio) deformability-induced lift acts towards channel center, while acting towards the wall for λ_{d-1}	X	O	X (not small λ_d)
Reversing streamlines created near a particle in confined flow	X	X	X
Reversing streamlines create repulsive particle-particle interactions	X	O	X
Trains of particles self-assemble due to particles interacting with the walls and with each other	O	O	X
Finite sized particles create a net secondary flow (i.e. particle-induced convection) at $R_p \gtrsim 2$	O	X	X
Particle equilibrium position is unaffected by the self-induced flow disturbance	O	O	X
Particle-induced convection scales with a^3 , U^2 and μ	O	X	O
Curving channels can be used to achieve shorter focusing length (due to the flow disturbance caused by Dean flow)	X	O	X
Curving channels can be used to achieve single stable equilibrium positions	O	O	X
Irregularities such as grooves, pillars, channel curvature etc. create considerable secondary flows	X	X	X
Four dominant modes of secondary flows for inertial flow deformation around pillars	O	X	X
Magnitude of the inertial flow deformation (secondary flow) induced by a structure	O	X	X

depends strongly on the structure size			
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List of symbols

a	Particle diameter
d_c	Length scale of change
D_{max}	Rotational diameter
F_L	Net lift
$F_{L,d}$	Deformability-induced lift
f_L	Lift coefficient
H	Channel dimension
h	Channel size in the dominant direction of particle migration
w	Channel size in the orthogonal direction of particle migration
L	Channel length
L_f	Channel length required for particles to reach lateral equilibrium positions
N	Number of particles
R	Radius of channel curvature
d	Distance between drop and center of the channel
d_s	Interparticle spacing
R_f	Lift to dean drag force ratio (F_L/F_D)
U	Mean channel velocity
ΔU	Velocity change
U_m	Maximum channel velocity
$\alpha=a/H$	Channel blockage ratio
$\dot{\gamma}$	Shear rate
λ	Fluid relaxation time
μ	Dynamic viscosity
μ_d	Dynamic viscosity of fluid inside a droplet
λ_d	Viscosity ratio
ρ	Density of fluid
ρ_p	Density of particle

σ	Surface tension
$\varphi = Na/L$	length fraction