Electronic supplementary material

The following description of the network based simulation is an adaption from the description in the ESI material of Zehnle *et al*. (Zehnle et al. 2012).

The network model can be seen in Fig 1. Tab. 1 describes the function of the individual fluidic elements depicted in Fig. 1. Fig. 2 shows one implementation of the timer structure. The figure highlights the implementation of the individual fluidic elements as described in Fig. 1 and Tab. 1.

Each fluidic element has 6 ports. These ports are used to transfer information between neighboring fluidic elements. Two or more ports are combined at one node.

- Pressure and flow rate is transferred via two hydraulic ports (black ports)
- The wetting state (wet / dry) at the beginning and the end of a fluidic element is transferred and received via a total of four digital port (blue ports)
- The analog input port (green port) receives the stimulus from the signal generator, in our case the analog port receives the rotational frequency of the system.

Any ports which are not connected are defined as „dry", since they are connected to ground.

For each fluidic element, the flow rate q can be calculated numerically from the pressure differential Δp between the ends of the element. The pressure across a radial element is defined as:

$$\Delta p(q) = p_{centrifugal} + p_{viscous}(q) + p_{inertial}(q) + p_{capillary} .$$

For an isoradial element the pressure differential is defined as:

$$\Delta p(q) = p_{Euler} + p_{viscous}(q) + p_{inertial}(q) + p_{capillary} .$$

The fill level and the resulting volume of liquid for each element is calculated by integrating the flow rate through the fluidic element:

$$V_{liquid}(t) = \int_0^t \dot{V}_{liquid}(t') \, dt'$$

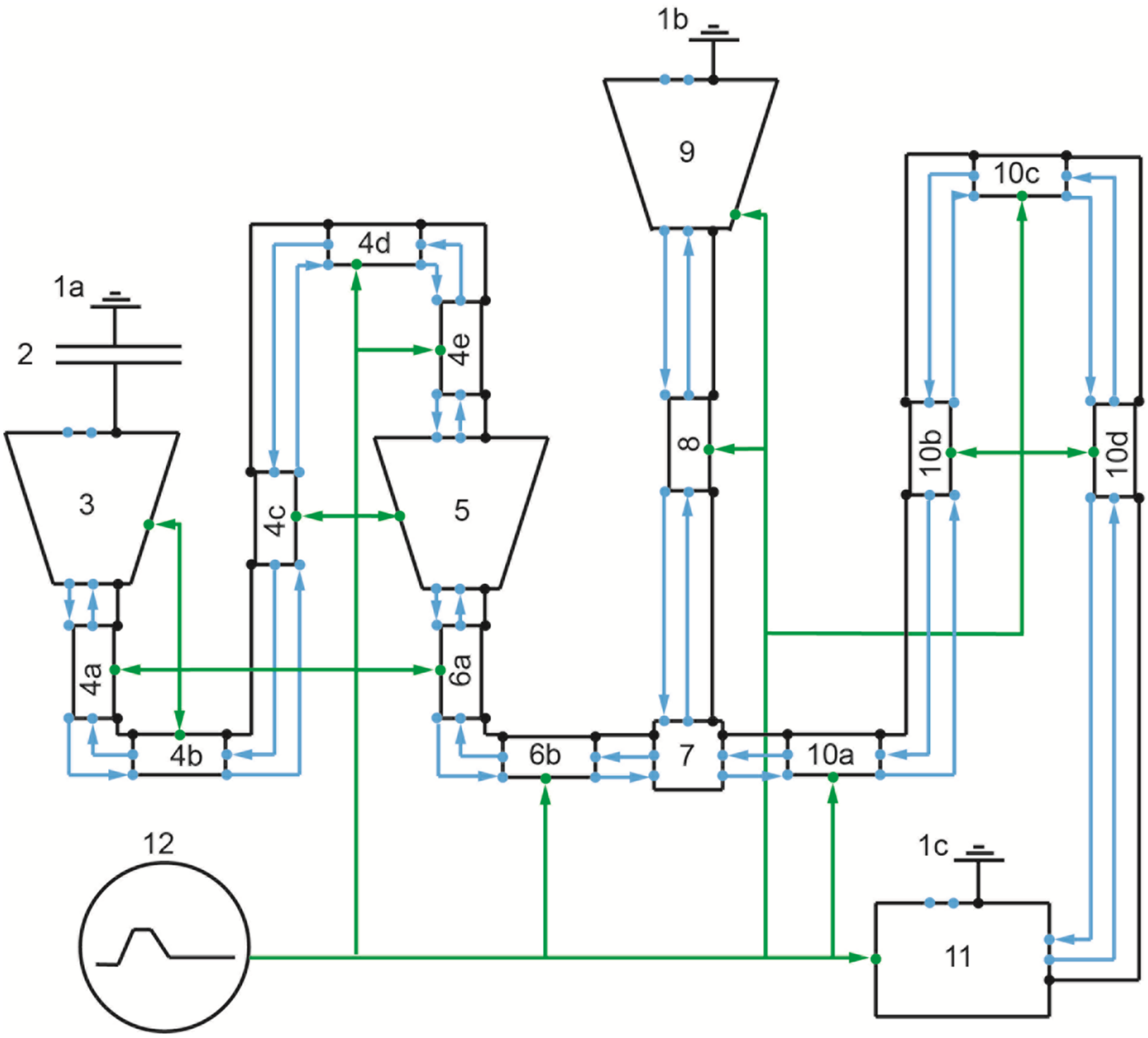

Fig. 1: Fluidic network as simulated in the network simulation.

Tab. 1: List of functional elements as modeled in the network simulation

No.	Name	Function
1a	Ground	Models ambient pressure; reference pressure: 1013 hPa
1b	Ground	Models the air vent in the inlet chamber; reference pressure: 1013 hPa
1c	Ground	Models the air vent in the collection chamber; reference pressure: 1013 hPa
2	Ideal gas	Fluidic capacitance: Models the enclosed air as an ideal gas
3	Pressure chamber 2	Models compression chamber 2 as a fluidic element with radial orientation
4a-4e	Timing channel	Models the timing channel as a series of channels with radial or tangential orientation
5	Pressure chamber 1	Models compression chamber 1 as a fluidic element with radial orientation
6a – 6b	Connection channel	Models the channel connecting compression chamber 1 with the inlet channel and the siphon channel as a series of radial and tangential channels
7	T-connector	Collapses all connected ports to nodes
8	Inlet channel	Models the timing channel as a fluidic element with radial orientation
9	Inlet chamber	Models the inlet chamber as a fluidic element with radial orientation
10a-10d	Siphon channel	Models the siphon channel as a series of radial and tangential channels
11	Collection chamber	Models the collection chamber as an element that takes up incoming liquid
12	Signal generator	Models the rotational frequency: It provides the stimulus for the fluidic elements as a piecewise linear function

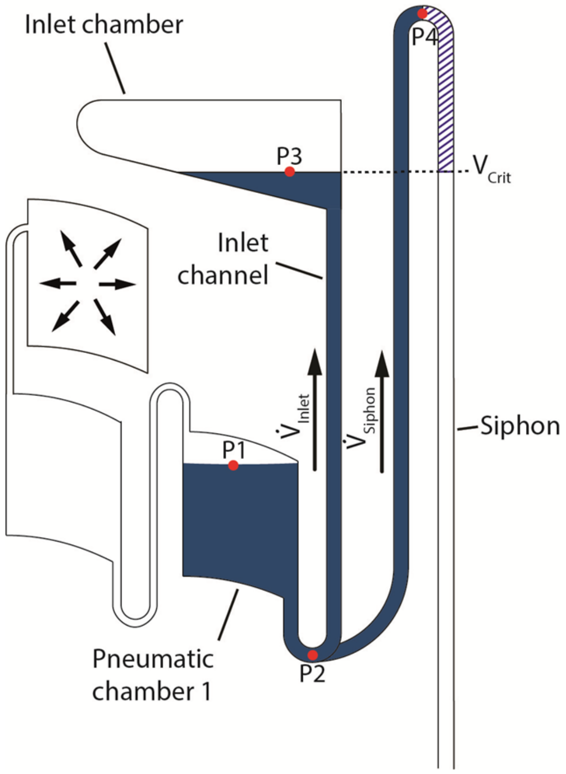

Fig. 2: *Illustration of siphon priming. The flow from compression chamber 1 is split into the siphon and the inlet channel. The siphon primes if a critical volume flows into the siphon. The volume that flows into the siphon is defined by equations 1-4 and can be calculated by numerical integration.*

The pressures at point P3 and P4 is the atmospheric pressure p_{atm}. Since the viscous dissipation of air in the timing channel can be neglected, the pressure at point P1 is equal to the pneumatic pressure in the pressure chamber p_{pneu}:

$$p_{P1}(V_{comp}) = p_{pneu}(V_{comp}) = \frac{p_{atm} V_0}{V_0 - V_{comp}} \qquad \text{(Eq. 1)}$$

The pressure at point P2 is then defined as:

$$p_{P2}(V_{comp}) = p_{P1}(V_{comp}) + p_{cent}(r_{P1}, r_{P2}) - p_{visc}(\dot{V}_{comp}, R_{P1\rightarrow P2}), \qquad \text{(Eq. 2)}$$

with $p_{cent}(r_{P1}, r_{P2})$ being the centrifugal pressure that accumulates between P1 and P2, $\dot{V}_{comp}$ being the total flow rate between P1 and P2 and $p_{visc}(\dot{V}_{comp}, R_{P1\rightarrow P2})$ being the viscous dissipation between P1 and P2:

$$p_{visc}(\dot{V}_{comp}, R_{P1\rightarrow P2}) = \dot{V}_{comp} \; R_{P1\rightarrow P2} \qquad \text{(Eq. 3)}$$

From the pressure difference between P2 and P3, and P2 and P4 it follows that:

$$p_{P1}(V_{comp}) + p_{cent}(r_{P1}, r_{P2}) - p_{visc}(\dot{V}_{comp}, R_{P1\to P2}) = p_{atm} + p_{cent}(r_{P2}, r_{P3}) - p_{visc}(\dot{V}_{Inlet}, R_{P2\to P3})$$

(Eq. 4)

$$p_{P1}(V_{comp}) + p_{cent}(r_{P1}, r_{P2}) - p_{visc}(\dot{V}_{comp}, R_{P1\to P2}) = p_{atm} + p_{cent}(r_{P2}, r_{P4}) - p_{visc}(\dot{V}_{Siphon}, R_{P2\to P4})$$

(Eq. 5)

If we insert the viscous pressure drop from eq. 3 into eq. Eq. 4 & 5 we receive:

$$\dot{V}_{Siphon} = \frac{p_{P1}(V_{comp}) + p_{cent}(r_{P1}, r_{P2}) - p_{visc}(\dot{V}_{comp}, R_{P1\to P2}) - p_{atm} - p_{cent}(r_{P2}, r_{P4})}{R_{P2\to P4}}$$

(Eq. 6)

$$\dot{V}_{Inlet} = \frac{p_{P1}(V_{comp}) + p_{cent}(r_{P1}, r_{P2}) - p_{visc}(\dot{V}_{comp}, R_{P1\to P2}) - p_{atm} - p_{cent}(r_{P2}, r_{P3})}{R_{P2\to P3}}$$

(Eq. 7)

According to the continuity equation for incompressible liquids, the flow rate between P1 and P2 has to be the same as the sum of flow rates in the inlet channel and the siphon:

$$\dot{V}_{comp} = \dot{V}_{Inlet} + \dot{V}_{Siphon} \qquad \text{(Eq. 8)}$$

The radial positions r_{P1}, r_{P3} and r_{P4}, as well as the fluidic resistance $R_{P2\to P4}$ are dependent on the fill level of the respective channels and chambers and thus can be expressed by a volume to height function in dependence of the total volume in the compression chamber V_{comp}. If this is taken into account, the total volume flowing into the siphon channel can be calculated by numerical integration of the differential equations Eq. 6, Eq. 7 and Eq. 8:

$$V_{siphon}(t) = \int_{t_{delay}}^{t} \dot{V}_{Siphon}\, dt' \qquad \text{(Eq. 9)}$$

The siphon primes if the volume $V_{siphon}(t)$ at any point in time becomes larger than V_{crit}. V_{crit} is the volume required to fill the siphon across the siphon crest and further so that centrifugal gravity causes transport of liquid to the outlet.

This condition is fulfilled if the meniscus in the siphon has passed the crest and subsequently moves to a position radially outwards of the liquid level in the inlet chamber (P3).

The volume in the siphon $V_{siphon}(t)$ can be calculated from Eq. 9 in combination with Eq. 6 to Eq. 8. Since the volume transported into the siphon depends on centrifugal acceleration as indicated by the terms $p_{cent}(r_{Pi}, r_{Pj})$, the rotational protocol can be used to control whether the siphon does or does not prime. A numerical solution of Eq. 9 for two different rotational protocols, one that inhibits and one that ensures siphon priming, is depicted in Fig. 3.

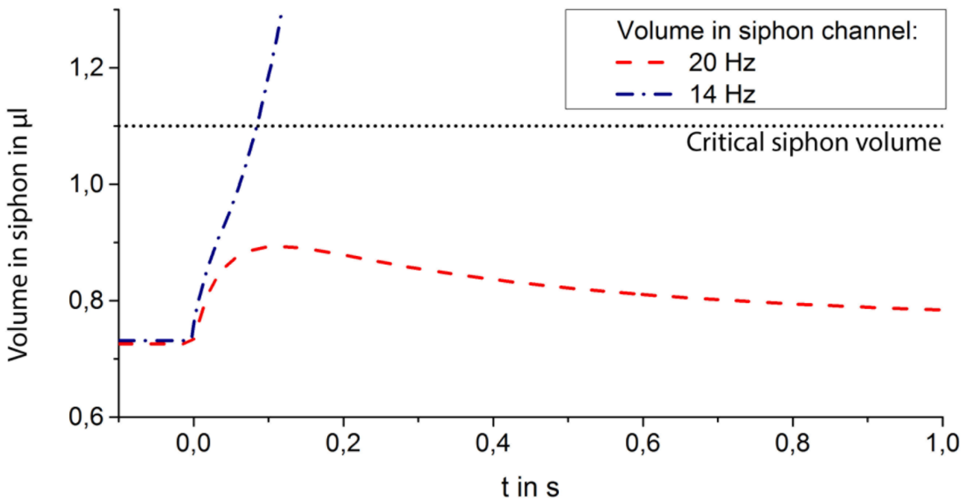

Fig. 3: *Simulation of siphon priming for timer #3 in the release on demand disk. The volume pumped into the siphon is described by Eq. 6-9 and depends on the splitting of flow between the inlet channel and the siphon channel, based on fluidic resistances and centrifugal pressures within the siphon channel and the inlet channel. The timer release is displayed at the time 0 s, the volume in the siphon channel quickly rises after timer release. If more than the critical volume is filled into the siphon, the siphon is primed and liquid is transferred to the collection chamber. At 14 Hz centrifugation, the siphon primes and liquid is transferred. At 20 Hz the centrifugal counter pressure from the siphon channel inhibits siphon priming and no liquid is transferred to the collection chamber.*

References

Zehnle, Steffen; Schwemmer, Frank; Roth, Günter; von Stetten, Felix; Zengerle, Roland; Paust, Nils (2012): Centrifugo-dynamic inward pumping of liquids on a centrifugal microfluidic platform. In: *Lab on a Chip* 12 (24), S. 5142–5145.