Electronic Supplementary Material (ESI) for Lab on a Chip.

Electronic Supplementary Information

Rational design of capillary-driven flows for paper-based microfluidics

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In this section we include some mathematical details related to the comparison of our (one-dimensional) model with a two-dimensional calculation recently reported, for the particular case of capillary imbibition in the presence of a sudden expansion.

In a recent work,[1] the capillary imbibition in porous media that undergoes a sudden expansion was theoretically studied by using potential flow with the elliptic coordinate system shown Fig. S1. The concentric elliptical curves are identified as η whereas hyperbolic curves are designated by its asymptotic angle ψ. The points $+a$ and $-a$ are the foci of the ellipses. Ellipses η correspond to isobaric lines. The liquid enters at $\eta = 0$, and the advancing front line is η_f. There is no flow across hyperbolic curves, hence they constitute the edges of the domain, named ψ_1 and ψ_2 respectively. Here we are interested in symmetrical domains with $\psi_2 = \pi - \psi_1$, thus opening angles are defined by $\beta = \pi - 2\psi_1$. The initial width is given by $w_0 = 2a\cos\psi_1$, and the distance covered by the fluid along flow axis is given by $l = a\sinh\eta_f$.

Also in Ref. 1, the pressure in the flow domain is considered to vary from atmospheric pressure $P_0 = P_{atm}$ at the entrance ($\eta = 0$) to capillary Laplace's pressure P_c at the liquid front ($\eta = \eta_f$). For these boundary conditions, flow rate is given by

$$Q(l) = \frac{hk}{\mu\,\eta_f(l)}\beta(P_0 - P_c) \tag{S1}$$

In our calculations we also consider an initial load R_0; therefore, introducing $P_0 = P_{atm} - QR_0$ and reordering, one has

$$Q(l) = \frac{hk/\mu \Delta P}{\frac{hk}{\mu}R_0 + \frac{1}{\beta}\eta_f(l)} \tag{S2}$$

In addition, by defining the dimensionless variables Q/hD and $hkR_0/\mu = l_0/w_0$, the resulting expression is

$$\frac{Q_{2D}}{hD} = \left[\frac{l_0}{w_0} + \frac{1}{\beta}\eta_f\right]^{-1} \tag{S3}$$

where η_f is given by,

$$\eta_f = \sinh^{-1}\left(2\cos\left(\frac{\pi-\beta}{2}\right)\frac{l}{w_0}\right). \tag{S4}$$

Finally we derive the flow rate predicted by our model for the opening hyperbolic shape of Fig. S1. Firstly one has to express the hyperbolic width as,

$$w(x) = w_0\sqrt{1 + \left(\frac{2}{tan\frac{\pi-\beta}{2}}\frac{x}{w_0}\right)^2} \tag{S5}$$

Then introducing this function into eqn. (5) and reordering yields,

$$\frac{Q_{1D}}{hD} = \left[\frac{l_0}{w_0} + \frac{\tan\left(\frac{\pi}{2}-\frac{\beta}{2}\right)}{2}\eta_l\right]^{-1} \tag{S6}$$

where η_l is given by,

$$\eta_l = \sinh^{-1}\left(\frac{2}{\tan(\pi/2-\beta/2)}\frac{l}{w_0}\right) \tag{S7}$$

The predictions of eqns. (S3) and (S7) are plotted in Fig. 5b, and the relative error $(Q_{1D} - Q_{2D})/Q_{2D}$ is reported in Fig. 5c.

1. E. M. Benner and D. N. Petsev, Phys Rev. E, 2013, 87, 033008

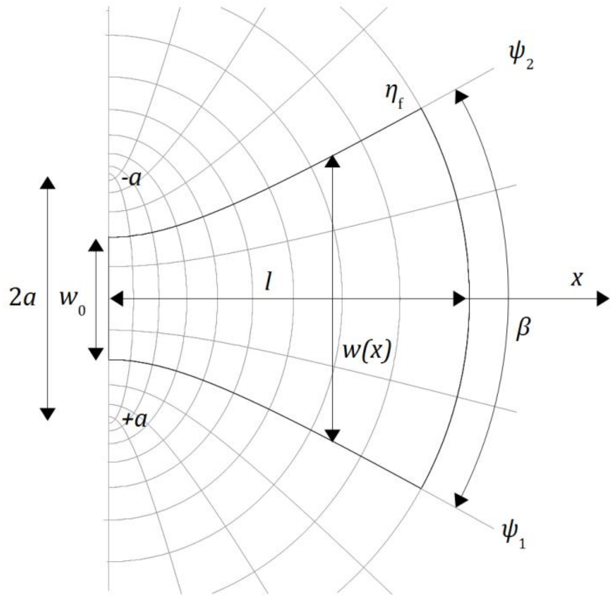

Fig S1 Schematic drawing of expanding porous media and elliptic coordinate system.