Electronic Supplementary Material (ESI) for Molecular BioSystems.

Supplementary Text T3

Given 2 contact maps M_1 and M_2, denote $x^r_{i,j}$ to be $M_r[i,j]$, the interaction frequency between loci *i* and *j* in the contact map M_r, $r = 1,2$. The Bayes Factor of loci *i* and *j*, denoted as *BFi,j*, is then:

$$(1)\ BF_{ij} = \frac{P(x^1_{i,j}\,,\, x^2_{i,j}\,|H_1)}{P(x^2_{i,j}\,,\, x^2_{i,j}\,|H_0)} = \frac{P(x^1_{i,j})\,P(x^2_{i,j})}{P(x^1_{i,j}\,,\, x^2_{i,j})}$$

and $P(x^1_{i,j}, \ldots\, x^r_{i,j})$ is the marginal likelihood of the data.

We assume here that corrected and standardized $x^r_{i,j}$ come from a Gaussian distribution with a Gamma prior, where both mean (μ) and variance (σ2) vary. Under this assumption the marginal likelihood can be written as:

$$(2)\ P(x^1_{i,j}, \ldots\, x^n_{i,j}) = \int \prod_{r=1}^{n} [\, p(x^r_{i,j}\,|\mu, \sigma^2)\,]\; p(\mu\,|\mu_0\,, \sigma_0^2)\, d\mu$$

Using a close form of a Normal-Gamma prior [1-2], Eq. 2 can be re-written as (notation used as in [2]):

$$(3)\ P(x^1_{i,j}, \ldots\, x^n_{i,j}) = \frac{\Gamma(\alpha_n)}{\Gamma(\alpha_0)} \frac{{\beta_0}^{\alpha_0}}{{\beta_n}^{\alpha_n}} \left(\frac{k_0}{k_n}\right)^{\frac{1}{2}} (2\pi)^{-\frac{n}{2}}$$

And

$$k_0 = \frac{2s^2}{\sigma_0^2};\ \ \beta_0 = \frac{1}{2}\sigma_0^2\ ;\ \alpha_0 = 1.0$$

$$k_n = k_0 + n;\ \ \beta_n = \beta_0 + \frac{1}{2}\left(\sum_{l=1}^{n} x^l_{i,j} - \bar{x}\right) + \frac{k_0 n(\bar{x} - \mu_0)^2}{2(k_0+n)};\ \alpha_n = \alpha_0 + \frac{n}{2}$$

Where $\bar{x}$ and s^2 are the mean and variance of $x^r_{i,j}$ respectively, μ_0 and σ_0^2 are the prior mean and variance (correspondingly) and Γ is the gamma function. Here, we have calculated separated μ_0 and σ_0^2 for *cis*, t*rans* and *self*- interactions, by taking the mean and variance of all the relevant interactions in M_1 and M_2, i.e:

$$(4)\ \ \mu_{0\_T} = mean(x^r_{i,j})\ ;\ \ \sigma^2_{0\_T} = variance(x^r_{i,j}),\ \ \textit{r=1,2 and the interaction between loci i and j is of type T (=self,cis,trans)}$$

The prior mean and variance in Eq. 2 are then selected based on the type of interaction (*cis, trans* or *self*) of loci *i* and *j* as calculated in Eq.4.

Finally, putting Eq. 1 - 3 together gives:

$$(5)\ BF_{ij} = \frac{\left(\frac{\Gamma(\alpha_1)}{\Gamma(\alpha_0)}\right)^2}{\frac{\Gamma(\alpha_2)}{\Gamma(\alpha_0)}} \frac{(2\pi)^{-1}}{(2\pi)^{-1}} \frac{\frac{{\beta_0}^{\alpha_0}}{{\beta_{1,1}}^{\alpha_1}} \frac{{\beta_0}^{\alpha_0}}{{\beta_{1,2}}^{\alpha_1}}}{\frac{{\beta_0}^{\alpha_0}}{{\beta_2}^{\alpha_2}}} \frac{\frac{k_0}{k_1}}{\left(\frac{k_0}{k_2}\right)^{\frac{1}{2}}} = \frac{\Gamma(\alpha_1)^2}{\Gamma(\alpha_2)} \frac{({\beta_0}^{\alpha_0})\ {\beta_2}^{\alpha_2}}{({\beta_{1,1}}^{\alpha_1})\ {\beta_{1,2}}^{\alpha_1}} \frac{\frac{k_0}{k_1}}{\left(\frac{k_0}{k_2}\right)^{\frac{1}{2}}}$$

Where $\beta_{1,r}$ is the β_1 value computed for $x^r_{i,j}$.

References

1. DeGroot MH: Optimal Statistical Decisions. New York: McGraw-Hill; 1970.
2. Murphy KP: **Conjugate Bayesian analysis of the Gaussian distribution**. Technical report. University of British Columbia; 2007.