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Review

Nanomechanics For MEMS: A Structural Design Perspective

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Online Supplementary Information

Extreme Value Theory

The extreme value theory (EVT) deals with the application of the statistics of extremes. [35,36] The EVT is a part of the more general theory of the "order statistics". By assuming x to be a random variable from a given population, $f(x)$ and $F(x)$ are the associated probability density function (*pdf*) and the cumulative distribution function (*cdf*), respectively. Also, assume that $\left(X_1, X_2 ..., X_n\right)$ is a random sample of size n from such population and rearrange the observations in increasing order $\left(X_{1:n}, X_{2:n} ..., X_{n:n}\right)$, with $X_{1:n} \leq X_{2:n} \leq ... \leq X_{n:n}$. The r-th order statistic is defined as the member $X_{r:n}$ of the ordered collection of size *n*. Depending on the application, one or more order statistics can critically affect functionality, performances or integrity of a complex system and the statistical properties of $X_{r:n}$ drive design and maintenance activities. Order statistics as a discipline can be used to determine the individual, joint, and conditional probability of one or more order statistics from the parent functions $f(x)$ and $F(x)$ of the population to which the sample belongs. Only the case of independent and identically distributed (IID) random samples is considered here. EVT has a more limited scope and is concerned with two order statistics only: the maximum and the minimum order statistics, i.e., the extremes defined as $X_{1:n} = \min\left(X_1, X_2 ..., X_n\right)$ and $X_{n:n} = \max\left(X_1, X_2 ..., X_n\right)$. In many applications, such as our failure problem, the statistics of either one of the two extremes is what matters. Weibull's theory is one of the first examples of EVT applied to structural engineering and allows inferring the strength of a full-size component or structure from the experimental data obtained in laboratory with smaller specimens of the material. In this paper we used EVT to estimate the probability of plastic failure of nanopillars and nanobuttons from the probability of "failure" (slip localization) on one generic slip plane. In a first order approximation, the distribution of the first order statistics (the minimum) is assumed to control the pillar strength. The distribution of random minima is expressed by the following *cdf* and *pdf*

$$F_{X_{1:n}}(x) = 1 - \left[1 - F(x)\right]^N , \qquad \text{(S1)}$$

$$f_{X_{1:n}}(x) = N\left[1 - F(x)\right]^{N-1} f(x) . \qquad \text{(S2)}$$

Eq.(S1) was used as basis for the statistical model discussed in the text. Remarkably, Eqs.(S1,S2) are not tied to any specific choice of parent distribution *f(x)*. The threshold of failure $x=\sigma$, the sample size *N* (i.e. the number of slip planes here) and the parent *pdf* and *cdf* need to be identified case by case and are left here for future work.