# Supplement Information

## Broadband Slow Light in One – Dimensional Logically Combined Photonic Crystals

G. Alagappan* and C. E. Png

Department of Electronics and Photonics, Institute of High Performance Computing, Agency for Science, Technology, and Research (A-STAR), Fusionopolis, 1 Fusionopolis Way, #16-16 Connexis, Singapore 138632

*Corresponding author: gandhi@ihpc.a-star.edu.sg

**Bloch mode decomposition technique**

Here, we will describe the details of the Bloch mode decomposition technique. This technique is used to calculate the Bloch mode expansion coefficient, $a_n$. [see Figure 2(e)]. The modes of PC1 are Bloch modes. Each of the Bloch mode can be identified with a pair of indices that denote the band index, $n$, and the wavevector, $k$. The Bloch mode, $\phi_{n,k}(x)$, of PC1 with frequency $\omega_{n,k}$, satisfy the 1D time – independent Maxwell's equation,

$$\left[\frac{d^2}{dx^2}+\frac{\omega_{n,k}^2}{c^2}\varepsilon_1(x)\right]\phi_{n,k}(x)=0 \qquad \text{[S1]},$$

where $\varepsilon_1(x)$ is the dielectric function of PC1. The Bloch modes of PC1 obey the orthogonalization condition,

$$\frac{1}{a}\int_{-a/2}^{a/2}\phi_{m,k'}^*(x)\varepsilon_1(x)\phi_{n,k}(x)dx=\delta_{mn,kk'} \qquad \text{[S2]}.$$

Let us expand the electric filed, $E(x)$, in the LCPC using the complete set of PC1's Bloch modes:

$$E(x)=\sum_{n,k}a_{n,k}\phi_{n,k}(x) \qquad \text{[S3]}.$$

This electric field obeys the 1D time – independent Maxwell's equation,

$$\left[\frac{d^2}{dx^2}+\frac{\omega^2}{c^2}\varepsilon(x)\right]E(x)=0 \qquad \text{[S4]},$$

where $\varepsilon(x)$ is the dielectric function of the LCPC. Expressing $\varepsilon(x)=\varepsilon_1(x)+\varepsilon_p(x)$, and using Eqns. S1 – S3, we can transform Eqn. S4, into a system of linear equations,

$$\frac{1}{a_s}\sum_{n,k} a_{n,k}\int_{-a_s/2}^{a_s/2}\phi^*_{m,k'}(x)\varepsilon_p(x)\phi_{n,k}(x)dx = \left(\frac{\omega^2_{m,k'}}{\omega^2}-1\right)a_{m,k'} \qquad [S5].$$

Writing $\frac{1}{a_s}\int_{-a_s/2}^{a_s/2}\phi^*_{m,k'}(x)\varepsilon_p(x)\phi_{n,k}(x)dx$ as $\left\langle \phi^*_{m,k'}\middle|\varepsilon_p\middle|\phi_{n,k}\right\rangle$, and re-arranging Eqn. S5, we arrive at the following symmetrical eigenvalue problem,

$$\sum_{n,k}\frac{\left\langle \phi^*_{m,k'}\middle|\varepsilon_p\middle|\phi_{n,k}\right\rangle + \delta_{mn,kk'}}{\omega_{m,k'}\omega_{n,k}}[a_{n,k}\omega_{n,k}] = \frac{c^2}{\omega^2}a_{m,k'}\omega_{m,k'} \qquad [S6],$$

with the eigenvalues $c^2/\omega^2$.

Before proceeding, let us examine the term, $\left\langle \phi^*_{m,k'}\middle|\varepsilon_p\middle|\phi_{n,k}\right\rangle$, in Eqn. S6. This term describes the coupling of PC1's Bloch mode due to the perturbation, $\varepsilon_p(x)$. Since, the period of $\varepsilon_p(x)$ is equal to $a_s = Ra$, the conservation of the translational symmetry requires $k-k'$ to be multiples of $g = 2\pi/a_s = 2\pi/Ra$. This means, different Bloch modes of PC1 will couple to each other, only if their wavevectors differ by multiples of *g*. [i.e., $\left\langle \phi^*_{m,k'}\middle|\varepsilon_p\middle|\phi_{n,k}\right\rangle \neq 0$, only when $k-k'$ is a multiple of *g*]. At such, it is useful to consider a folded band structure of PC1 [see the main text for the details]. In the folded band structure, the wavevectors of the adjacent bands differ exactly by *g*.

For a given wavevector, *k*, within the first BZ of the LCPC, the Bloch modes in the same folded cannot couple to each other. Only modes with the same *k*, but in different folded bands can couple. Therefore, for a given *k*, we can re-write Eqn. S6 using a single index subscript,

$$\sum_{n}\frac{\left\langle \phi^*_{m}\middle|\varepsilon_p\middle|\phi_{n}\right\rangle + \delta_{mn}}{\omega_{m}\omega_{n}}[a_{n}\omega_{n}] = \frac{c^2}{\omega^2}a_{m}\omega_{m} \qquad [S7].$$

Here, $n$ is the index of the folded band. Eqn. S7 can be written in a matrix form as $\hat{B}\vec{v} = [c^2 / \omega^2]\vec{v}$, where $\vec{v} = [a_1\omega_1 \quad a_2\omega_2 \quad .. \quad ..]^T$. Inverting this matrix equation we have,

$$\hat{B}^{-1}\vec{v} = [\omega^2 / c^2]\vec{v} \qquad [S8].$$

Solving the eigenvalue problem in Eqn. S8, the expansion coefficients [$a_n$] can be found. One can also use Eqn. 8 to obtain the band structure of the LCPC. Please take note that, if we express both $\phi_n$ and $\varepsilon_p$ in the plane wave basis (i.e., using a Fourier series), then Eqn. S8 will revert to the standard equations that describes the well-known plane wave expansion technique [17]. However, the information on $a_n$ will be lost in the plane wave basis.