Electronic Supplementary Material (ESI) for Nanoscale.

The original set of equations can be written as:

1.
$$\begin{cases} v = c_0 - \frac{3}{2}n - b \\ \frac{\partial n}{\partial t} = -2k_1 n^2 + 2k_2 v + D_1 \Delta n \\ \frac{\partial v}{\partial t} = k_1 n^2 - (k_2 + k_3)v + k_4 b + D_2 \Delta v \\ \frac{\partial b}{\partial t} = D_3 \Delta b + k_3 v - k_4 b \end{cases}$$

Using the time derivative of first equation to rewrite the second one yields

2.
$$\begin{cases} \frac{3}{2}n = c_0 - b - v \\ \frac{\partial 3/2n}{\partial t} = -\frac{\partial b}{\partial t} - \frac{\partial v}{\partial t} = -3k_1 n^2 + 3k_2 v + 3/2 D_1 \Delta n \\ \frac{\partial v}{\partial t} = k_1 n^2 - (k_2 + k_3)v + k_4 b + D_2 \Delta v \\ \frac{\partial b}{\partial t} = D_3 \Delta b + k_3 v - k_4 b \end{cases}$$

Rewriting the second equation with the right hand sides of the third and fourth ones and using the Laplacian $3/2\Delta n = -\Delta b - \Delta v$ of first equation yields:

3.
$$\begin{cases} \frac{3}{2}n = c_0 - b - v \\ -k_1 n^2 + k_2 v - D_2 \Delta v - D_3 \Delta b = -3k_1 n^2 + 3k_2 v - D_1 \Delta b - D_1 \Delta v \\ \frac{\partial v}{\partial t} = k_1 n^2 - (k_2 + k_3)v + k_4 b + D_2 \Delta v \\ \frac{\partial b}{\partial t} = D_3 \Delta b + k_3 v - k_4 b \end{cases}$$

The $2k_1 n^2$ term of the second equation can then be isolated and substituted back into the third one which then gets multiplied by $2(3D_2 - D_1 - 2D_3)$ whereas the fourth is multiplied by $2(D_3 - D_1)$.

4.
$$\begin{cases} \frac{3}{2}n = c_0 - b - v \\ 2k_1 n^2 = 2k_2 v + (D_2 - D_1)\Delta v + (D_3 - D_1)\Delta b \\ 2(3D_2 - D_1 - 2D_3)\frac{\partial v}{\partial t} = (3D_2 - D_1 - 2D_3)[(3D_2 - D_1)\Delta v + (D_3 - D_1)\Delta b - 2k_3 v + 2k_4 b] \\ 2(D_3 - D_1)\frac{\partial b}{\partial t} = 2(D_3 - D_1)D_3 \Delta b + 2(D_3 - D_1)k_3 v - 2(D_3 - D_1)k_4 b \end{cases}$$

Adding the fourth equation to the third one and writing $X = (6D_2 - 2D_1 - 4D_3)v + (2D_3 - 2D_1)b$ finally leads to:

5.
$$\begin{cases} \frac{3}{2}n = c_0 - b - v \\ 2k_1 n^2 = 2k_2 v + (D_2 - D_1)\Delta v + (D_3 - D_1)\Delta b \\ \frac{\partial X}{\partial t} = \frac{3D_2 - D_1}{2}\Delta X + k_3 \frac{3D_3 - 3D_2}{3D_2 - D_1 - 2D_3} X + 2(3D_2 - 3D_3)[k_4 + k_3 \frac{D_3 - D_1}{3D_2 - D_1 - 2D_3}]b \\ \frac{\partial b}{\partial t} = D_3 \Delta b + \frac{k_3}{6D_2 - 2D_1 - 4D_3} X + [\frac{2D_1 - 2D_3}{6D_2 - 2D_1 - 4D_3} - k_4]b \end{cases}$$

Term by term identification with the following Turing reaction-diffusion system

6. $$\begin{cases} \frac{\partial X}{\partial t} = a_1 \Delta X + a_2 X + a_3 b \\ \frac{\partial b}{\partial t} = a_4 \Delta b + a_5 X + a_6 b \end{cases}$$

leads to:

7. $$\begin{cases} a_1 = \frac{3D_2 - D_1}{2} \\ a_2 = k_3 \frac{3D_3 - 3D_2}{3D_2 - D_1 - 2D_3} \\ a_3 = 2(3D_2 - 3D_3)[k_4 + k_3 \frac{D_3 - D_1}{3D_2 - D_1 - 2D_3}] \\ a_4 = D_3 \\ a_5 = \frac{k_3}{6D_2 - 2D_1 - 4D_3} \\ a_6 = \frac{2D_1 - 2D_3}{6D_2 - 2D_1 - 4D_3} - k_4 \end{cases}$$