High performance cascaded PDMS micromixer based on split-and-recombination flows for lab-on-a-chip applications

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Electronic Supplementary Information

3.2 Simulation results

The simulation results for mixing modules that are made up of two and three mixing units are shown in Figure 1S-a a and Figure 1S-b. In each case, the distribution of concentration at the outlet cross-section is shown. The results show that at a high flow rate of 50µl/min, the mixing efficiency of a mixing module that is built from two and three mixing units is 83% and 92% respectively. The mixing efficiency for various flow rates between 2µl/min to 100µl/min is shown in Figure 4-d of the paper. The results show that the cascading of two micromixers can achieve full mixing for high Reynolds number of $R_e < 13.5$ corresponding to the flow rate up to $Q=100\mu$ l/min. One could conclude that the proposed mixer configuration enables very high mixing efficiency even at high flow rates.



Figure 1S. Simulation of cascading of several mixing units for achieving better mixing performance at a high flow rate of 50μ l/min; (a) two units (η =82.5%) and (b) three units (η =91.6%).

5.2 Mixing testing of acid and base

The efficiency is estimated based on the mixing of equal amounts of strong acid and strong base to yield a mixture with a pH of 7. As mixing takes place, the pH value of acid (inlet#1) increases from 2, while on the other side the pH value of base (inlet#2) of the mixer decreases from 12. Complete mixing is considered when the pH of the mixture reaches to the value of 7. A typical measured pH variation that was used to estimate the efficiency is shown in black in Fig. 2S-a and b corresponding to a flow rate of 200µl/min. This graph is denoted as the function p(x) where x is the normalized outlet channel's width changing between 0 and 1. The hatched area represents the actual mixing. In Fig. 2S-a and b red and green lines correspond to no-mixing and fullmixing situations and are denoted by the functions $p_1(x)$ and $p_2(x)$, respectively.

In order to obtain the variation of efficiency along the outlet cross-section, the mixing efficiency in each outlet branch (4 branches) was obtained and then was plotted against the channel widths at the mid points of the branches as x=0.125, x=0.375, x=0.625, and x=0.875.

Considering equal flow rates from all the outlets, the following equations were used to calculate the efficiency in each branch:

$$\eta_{x=0.125} = \frac{\text{area corresponding to actual mixing } (0 \le x \le 0.25)}{\text{area corresponding to full mixing } (0 \le x \le 0.25)} = \frac{\int_0^{0.25} |p_1(x) - p(x)| \, dx}{\int_0^{0.25} |p_2(x) - p_1(x)| \, dx}$$
(1-1)

$$\eta_{x=0.375} = \frac{area \ corresponding \ to \ actual \ mixing \ (0.25 \le x \le 0.5)}{area \ corresponding \ to \ full \ mixing \ (0.25 \le x \le 0.5)} = \frac{\int_{0.25}^{0.5} |p_1(x) - p(x)| \ dx}{\int_{0.25}^{0.5} |p_2(x) - p_1(x)| \ dx}$$
(1-2)

$$\eta_{x=0.625} = \frac{area \ corresponding \ to \ actual \ mixing \ (0.5 \le x \le 0.75)}{area \ corresponding \ to \ full \ mixing \ (0.5 \le x \le 0.75)} = \frac{\int_{0.5}^{0.75} |p_1(x) - p(x)| \ dx}{\int_{0.5}^{0.75} |p_2(x) - p_1(x)| \ dx}$$
(1-3)

$$\eta_{x=0.875} = \frac{\text{area corresponding to actual mixing } (0.75 \le x \le 1)}{\text{area corresponding to full mixing } (0.75 \le x \le 1)} = \frac{\int_{0.75}^{1} |p_1(x) - p(x)| \, dx}{\int_{0.75}^{1} |p_2(x) - p_1(x)| \, dx}$$
(1-4)

The area corresponding to actual mixing and the area corresponding to full mixing for the first branch (where $0 \le x \le 0.25$), are shown in Figure 2S-a-1 (hatched area) and Figure 2S-a-2 (grey area), respectively.

Furthermore, in order to calculate the average efficiency of the proposed micromixer at each flow rate, the following equation has been used by changing the integral limits:

$$\eta_{avr} = \frac{\text{area corresponding to actual mixing } (0 \le x \le 1)}{\text{area corresponding to full mixing } (0 \le x \le 1)} = \frac{\int_0^1 |p_1(x) - p(x)| \, dx}{\int_0^1 |p_2(x) - p_1(x)| \, dx}$$
(2)

The area corresponding to actual mixing is shown by the hatched area in Figure 2S-b-1 and the area showing the full mixing is depicted in grey in Figure 2S-b-2. The ratio between these two areas yields the average efficiency, η_{avr} , that is formulated in Eq. 2. Figure 2S-c shows the value of efficiency in each branch with is plotted against the outlet width and the value of average efficiency.



Figure 2S. Typical graph of pH measurements used for the estimation of mixing efficiency in experimental test at the flow rate of 200µl/min, (a) Calculation of efficiency in branch #1, (a-1) hatched area corresponding to actual mixing, (a-2) grey area corresponding to full mixing; (b) Calculation of the average mixing efficiency of the micromixer, (b-1) hatched area corresponding to actual mixing; (b-2) grey area corresponding to full mixing; (c) variation of mixing efficiency in branches and the average efficiency of the proposed micromixer.

To verify the results obtained in experiments (section 5.2), the same layout was simulated under different flow conditions. Table 1S shows the value of normalized flow rate that is passing through each branch for a flow rate of 20μ l/min. The simulation shows the flows passing through four branches are almost equal with a maximum variation of 2.3%.

Branch No.	Normalized	Ideal normalized flow rate	error
	Flow rate	that should pass	
1	0.2469	0.25	1.2%
2	0.2558	0.25	2.3%
3	0.2480	0.25	0.8%
4	0.2493	0.25	0.3%

Table 1S: The simulated value of flow-rate passing through each branch

Figure 3S-a shows a simulation of mixing while the outlet is branched into four branches. The simulation results for four different flow rates (10µl/min, 40µl/min, 80µl/min, and 150µl/min) are shown in Figure 3S-b. In this figure, the variation of mixing efficiency across the channel width is shown for each flow rate. As it can be seen, there is a maximum of 4.1% difference between the efficiency obtained in simulation and experiment. Furthermore, the value of average efficiency, η_{avr} , for each flow rate is provided in this figure. Comparing the results show, for example, for the flow rate of 150µl/min, the difference between the experimental and predicted average efficiency, η_{avr} , is less than 1.9%.



Figure 3S: The simulated results of the mixing experiments with four branches; (a) Mixing of acid and base at the flow rate of 80μ /min; (b) The variation of efficiency at the outlet cross-section and the average efficiency, η_{avr} , for each flow rate.