

## Supplementary information

### Fitting Procedure of extended Gent model

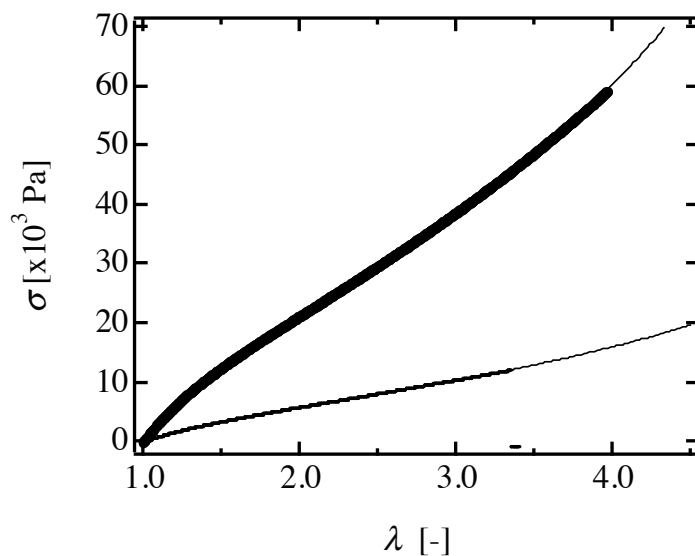
In order to estimate  $\lambda_{\max}$  qualitatively, we utilized the extended Gent model, which was proposed in our previous study to predict the biaxial stress-elongation behavior of the 20k Tetra-PEG gel. The extended Gent model has no molecular origin and the phenomenological nature, and predicts the uniaxial stress ( $\sigma$ )-elongation ( $\lambda$ ) relationship as

$$\sigma = \frac{C_1(\lambda - \lambda^{-2})}{\left(1 - \frac{\lambda^2 + 2\lambda^{-1} - 3}{\lambda_{\max}^2 + 2\lambda_{\max}^{-1} - 3}\right)} + C_2(1 - \lambda^{-3}) \quad (1)$$

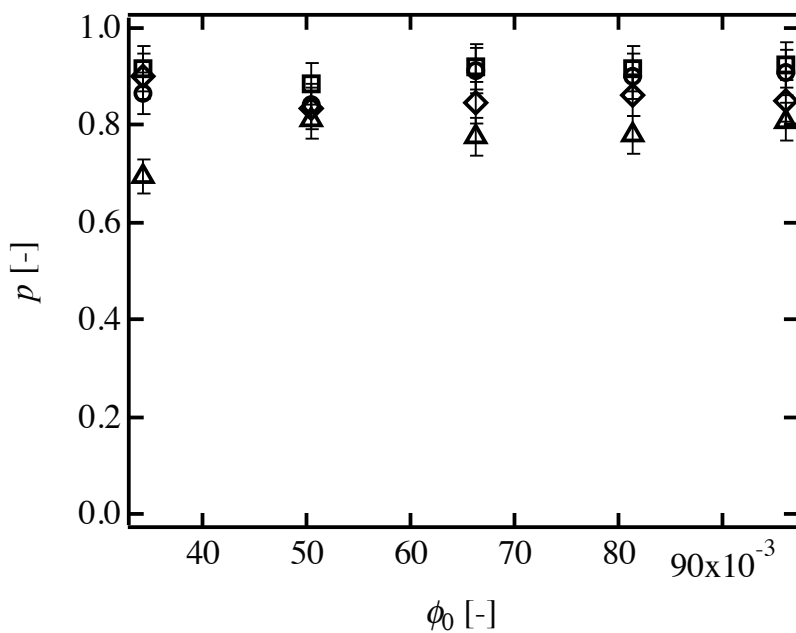
, where  $C_1$ ,  $C_2$ , and  $\lambda_{\max}$  are the fitting functions. The  $C_1$  and  $C_2$  have the relationship with elastic modulus ( $G$ ) as  $G = C_1 + C_2$ . It was suggested that  $C_1$  is almost the same value of  $G$ ,  $C_2$  is related to the stress coupling of different axes, and  $\lambda_{\max}$  is related to the ultimate elongation ratio.

In this study, we assumed that  $W$  of the 5k, 10k, and  $p$ -tuned Tetra-PEG gels are also predicted by the extended Gent model, and estimated the fitting parameters from the uniaxial stretching measurement. Because the uniaxial stretching is only a deformation among physically accessible deformations, the estimation of  $W$  function from the uniaxial stretching has ambiguity. However, we believe that the estimation of  $\lambda_{\max}$  is possible due to the following reason.

The extended Gent model predicts  $\sigma$  as the sum of two terms including  $\sigma_1$  and  $\sigma_2$ , of which prefactors are  $C_1$  and  $C_2$ , respectively. Supplementary Figure 6 shows the computation results of  $\sigma_1/\sigma_2$  against the elongation ratio in the  $x$  direction ( $\lambda$ ) with  $C_1/C_2 = 20$  and 100. As clearly shown in Supplementary Figure 6, the contribution of  $\sigma_2$  is extremely smaller than that of  $\sigma_1$  in uniaxial stretching than that in other deformations. Thus, practically  $\sigma_2$  has little effect on the estimation of  $\lambda_{\max}$  from the uniaxial stretching. In addition, the fit results of the 20k Tetra-PEG gel from the uniaxial stretching were similar to that from the biaxial stretching. Therefore, we decided to utilize the extended Gent model to predict  $\lambda_{\max}$  of the 5k, 10k and  $p$ -tuned Tetra-PEG gels. The fit of the extended Gent model worked well, and  $\lambda_{\max}$  was successfully estimated from the fit. It should be noted that the precise estimation of  $C_2$  is impossible from the uniaxial stretching, and we have to perform the biaxial stretching.

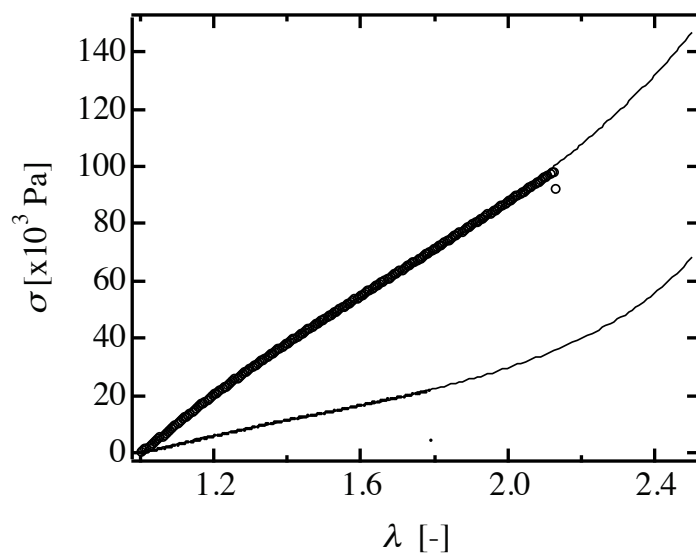


**Supplementary Figure 1. Stress-elongation relationship of the p-tuned Tetra-PEG gels. Thick and thin symbols show the results of  $p = 0.89$  and  $0.61$ , respectively.**

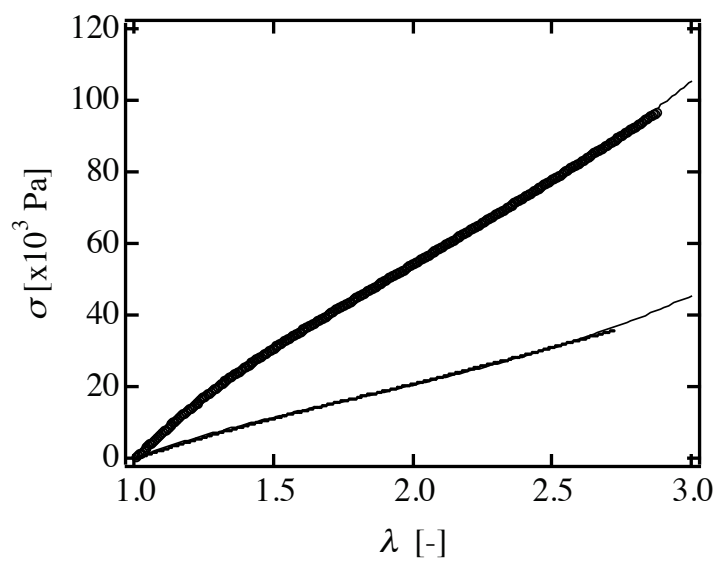


**Supplementary Figure 2. The reaction efficiency ( $p$ ) as a function of initial polymer volume fraction in the 5k, 10k, and 20k Tetra-PEG gels (5k, rhombus; 10k, circle; 20k, square).**

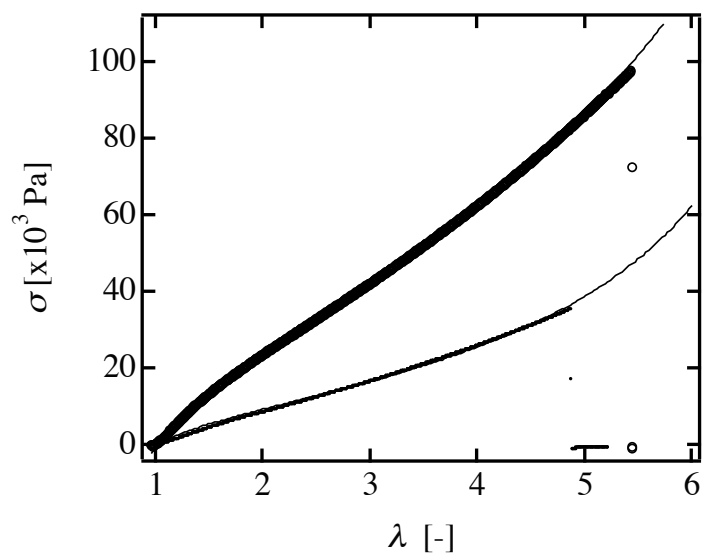
(a)



(b)

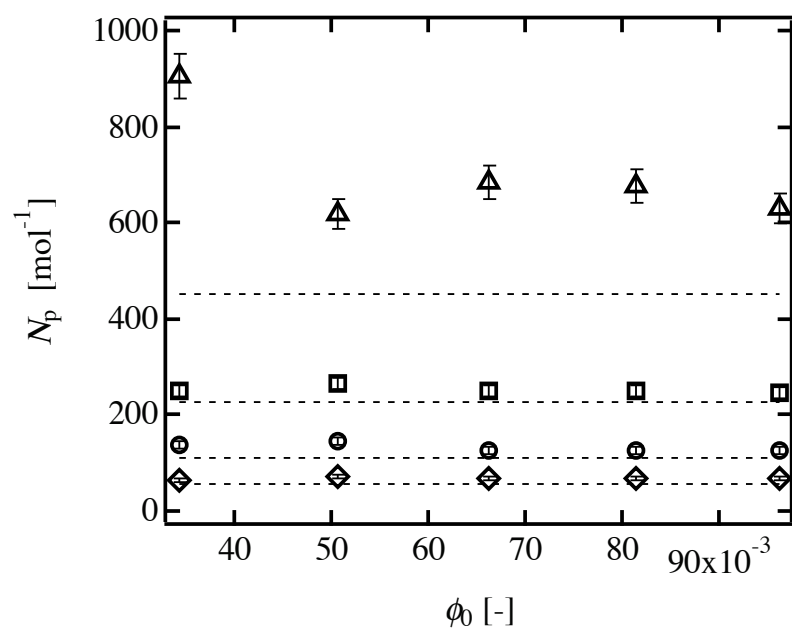


(c)

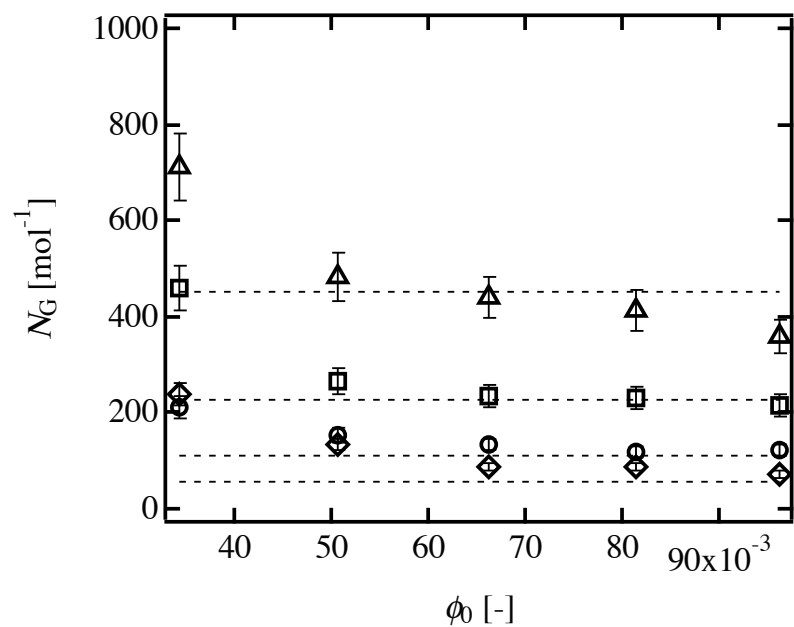


**Supplementary Figure 3. Stress-elongation relationship of the 5k (a), 10k (b), and 20k (c) Tetra-PEG gels. Thin and thick symbols show the results of  $\phi_0 = 0.051$  and 0.096, respectively.**

(a)

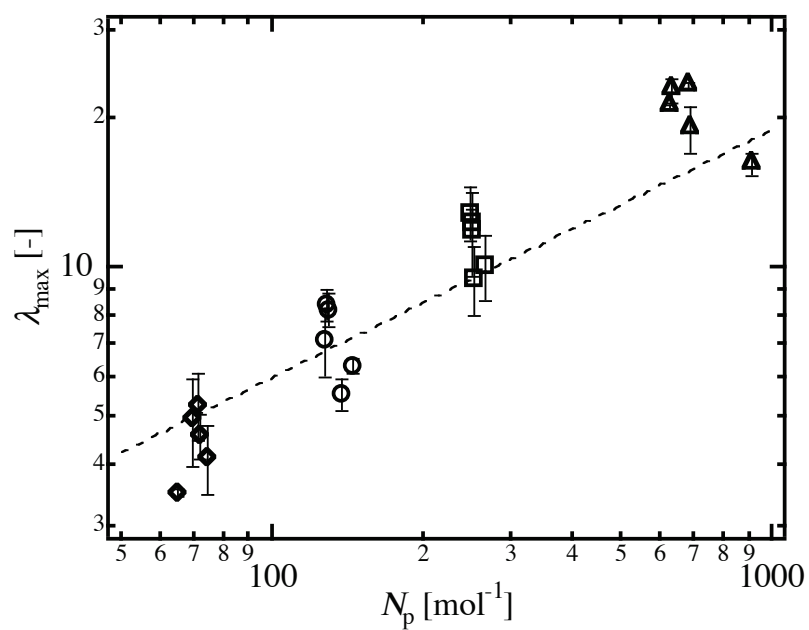


(b)

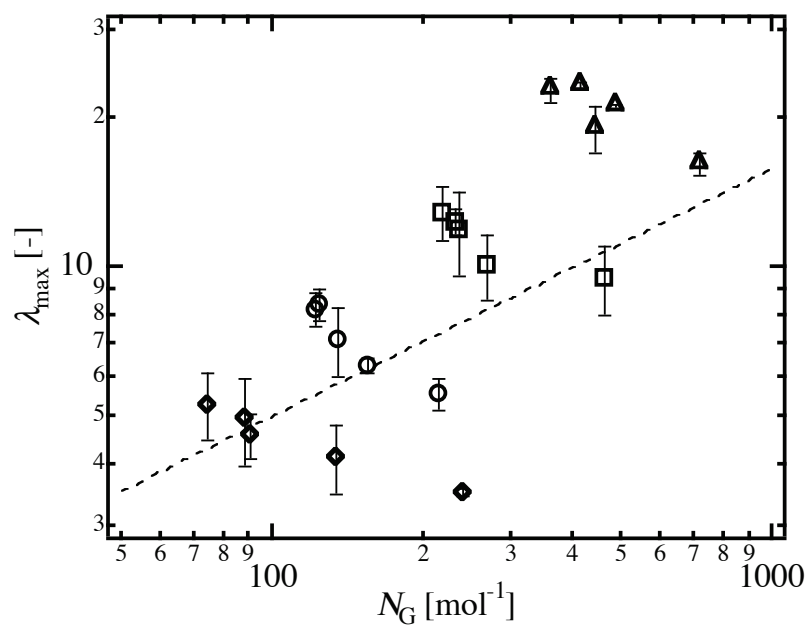


Supplementary Figure 4. (a)  $N_p$  and (b)  $N_G$  of 5k, 10k and 20k Tetra-PEG gels (5k, rhombus; 10k, circle; 20k, square).

(a)

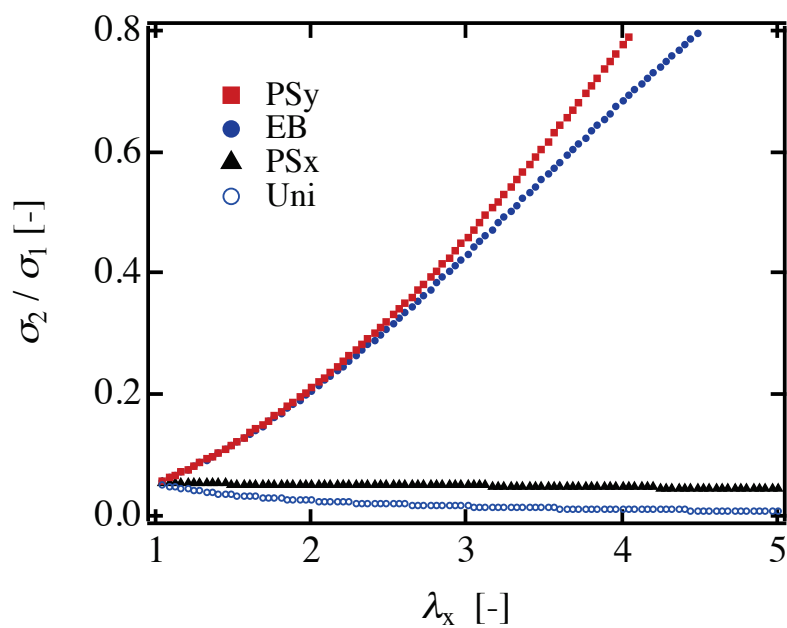


(b)

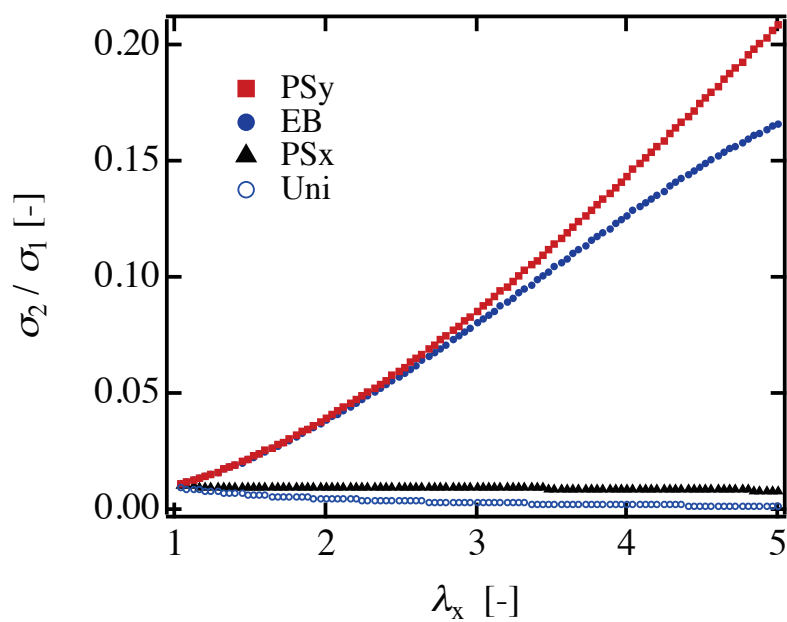


**Supplementary Figure 5.**  $\lambda_{\max}$  as a function of (a)  $N_p$  and (b)  $N_G$  in 5k, 10k and 20k Tetra-PEG gels (5k, rhombus; 10k, circle; 20k). The dotted line is the guide showing the relationship,  $\lambda_{\max} \sim N^{1/2}$ .

(a)



(b)



**Supplementary Figure 6.** The stress ratio  $\sigma_2/\sigma_1$  as a function of the elongation ratio in the x direction ( $\lambda_x$ ) under the equi-biaxial stretching (EB), pure shear in the stretching direction (PSx), in the constrained direction (PSy), and uniaxial stretching deformation (Uni). (a)  $C_1/C_2 = 20$ , (b)  $C_1/C_2 = 100$ .