Appendix

$$E_{ZGNS}(k) = \pm t\sqrt{1 + 4\cos\left(\frac{3k_x a_{cc}}{2}\right)\cdot\cos\left(\frac{2\pi\nu - \varphi}{2n}\right) + 4\cos^2\left(\frac{2\pi\nu - \varphi}{2n}\right)}$$

we define $\alpha = \dfrac{2\pi\nu - \varphi}{2n}$ **where** φ =0 for zigzag ZGNS and ν =1,2,....,2n(even)

IF $n \in N$ then, $\alpha = \dfrac{2\pi(2n)}{2n} = 2\pi$ **and** $\cos\alpha = \cos 2\pi = 1$

$$E_{ZGNS}(k) = \pm t\sqrt{1 + 4\cos\left(\frac{3k_x a_{cc}}{2}\right) + 4}$$

by applying taylor series: $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} \approx 1 - \dfrac{x^2}{2}$ then we get

$$E_{ZGNS} = \pm t\sqrt{5 + 4\left(1 - \frac{\left(\frac{3k_x a_{cc}}{2}\right)^2}{2}\right)}$$

$$E_{ZGNS} = \pm t\sqrt{9 + \frac{5}{2}\cdot\left(\frac{9k_x a_{cc}}{4}\right)}$$

$$E_{ZGNS} = \pm 3t\sqrt{1 + \frac{5}{8}k_x^2 a_{cc}^2}$$