Electronic Supplementary Material (ESI) for RSC Advances.

ELECTRONIC SUPPLEMENTARY INFORMATION

1. NUMERICAL METHOD

In this section, further details of the numerical are provided, in particular those concerning discretization and numerical implementation.

1.1 FLUID SOLVER

A second-order projection method is employed[1] in which an intermediate velocity $\boldsymbol{u}^*$ is computed,

$$\rho\frac{u^* - u^n}{\Delta t} = -\rho(u \cdot \nabla u)^{n+1/2} - \nabla p^{n-1/2} + \frac{\mu}{2}\left(\nabla_h^2 u^* + \nabla_h^2 u^n\right) + F^{n+1/2},$$

and the boundary conditions are given by

$$u^*|_{\delta\Omega} = u^{n+1}_{b}.$$

The viscous term has been split using a Crank-Nicolson scheme, resulting in a Helmholtz equation, and is solved by an alternate directions implicit (ADI) method. The advection term is computed using an Adams-Bashforth method:

$$(u \cdot \nabla u)^{n+1/2} = \frac{3}{2}(u \cdot \nabla_h u)^n - \frac{1}{2}(u \cdot \nabla_h u)^{n-1}.$$

The intermediate velocity field $\boldsymbol{u}^*$ does not satisfy the divergence free condition. A pseudo-pressure Φ is computed which is used to update the pressure and velocity so that it does satisfy divergence-free conditions:

$$\nabla_h^2 \phi^{n+1} = \frac{\nabla^2 u^*}{\Delta t}, n \cdot \nabla \phi^{n+1}|_{\delta\Omega} = 0.$$

The equation for the pseudo-pressure Φ is a Poisson equation with Neumann boundary conditions.

The velocity and pressure are then updated using:

$$u^{n+1} = u^* - \Delta t \nabla^2 \phi^{n+1},$$

$$p^{n+1} = p^{n-1/2} - \frac{v}{2}(\nabla^2 u^*) + \phi^{n+1}.$$

1.2 MEMBRANE DEFORMATION

This model is implemented numerically by first solving for the surface deformation gradient tensor $\boldsymbol{A}$ using only the current deformed shape of the membrane and the undeformed shape:

$$A \cdot \frac{\partial \bar{X}}{\partial \xi} = \frac{\partial X}{\partial \xi}, A \cdot \frac{\partial \bar{X}}{\partial \eta} = \frac{\partial X}{\partial \eta}, A \cdot \bar{n} = 0\ ,$$

η and ξ are local parametric coordinates defined over each triangle of the membrane grid.

The components of $\boldsymbol{A}$ are averaged for each element sharing the node to obtain a smoother distribution using weighted averages corresponding to the angle of each element attached to the node. Finally, the forces on each element can be computed by use of a line integral

$$\bar{\Delta} f = \frac{1}{S_n} \oint_C [b \cdot \tau] d\bar{l} ,$$

where S_n is the area enclosed by the contour C and $b = t \times n$ is the cross-product of the unit tangential vector along the contour and the surface unit normal vector.

1.3 Boundary conditions

The inlet and outlet velocities are the analytical solution for a flow in a square duct. For a rectangular channel of width W and of height H, with corresponding Cartesian coordinates y and z centered at the channel center, flowing in direction of increasing x, the velocity is given by $u = (u_x, 0, 0)$ [6], where

$$\frac{u_x}{K} = \left(\frac{H^2}{4} - z^2\right) + \sum_{n=1}^{\infty} B_n \cosh\left(\frac{2b_n y}{l_z}\right) \cos\left(\frac{2b_n z}{l_z}\right),$$

$$K = -\frac{1}{2\mu}\frac{dp}{dx},\ b_n = \frac{(2n-1)\pi}{2},\ and\ B_n = \frac{(-1)^n H^2}{b_n^3 \cosh\left(\frac{b_n W}{H}\right)}.$$

This infinite series is truncated at n = 50 in the current implementation. This provides 7 significant digits of accuracy. Care was taken that cells never approach the inlets or outlets in our simulations.

2. Validation

The numerical technique has been validated against previous results by simulating a spherical capsule in an infinite shear flow and comparing the deformation at steady-state against results from small deformation theory [2], boundary integral methods [3,4] and Immersed Boundary methods [5]. A capsule of radius R is deformed in pure shear flow placed in a domain of size 15R x 15R x 15R with a grid resolution of 135 x 135 x 135. The sphere consists of 5,120 triangular elements and 2,562 vertices. The system is then evolved until the capsule deformation reaches a steady state. When the capillary number (Ca) is small, results from the simulations can be compared to solutions from linear theory. For any capsule, the deformation at steady state is then given by $D_{12} = \frac{52+\nu}{41+\nu} Ca$, where the Poisson ratio ν is 0.5 for for a Neo-Hookean cell. D_{12} is the Taylor deformation ratio, given by $D_{12} = \frac{L-B}{L+B}$, where L and B are the major and minor axes respectively. At larger capillary numbers, results from small deformation theory are no longer accurate. The results from the present study (table 1) are similar to those obtained by others, and match most closely those obtained using the immersed boundary method.

Ca	Present	DB	RP	L1	SD
0.0125	0.084	0.083			0.078
0.025	0.162	0.162	0.16	0.15	0.156
0.05	0.275	0.278	0.27	0.27	0.313
0.1	0.391	0.392	0.39	0.40	
0.15	0.458	0.460		0.47	
0.2	0.492	0.496	0.5	0.52	

Table 1: Comparison of results from present work, Doddi and Bagchi (2008) (DB), Lac. et al. (2004) (L1), Ramanujan and Pozrikidis (1998) (RP) and small deformation theory (SD). Results presented are taken from Doddi and Bagchi (2008).

3. COMPARISON WITH EXPERIMENTAL RESULTS

To demonstrate the capabilities of our numerical approach, a comparison with experiments performed using a microfluidic cross-slot device that used viscoelasticity to focus the cells is presented in figure 1. We employed an upper-convective Maxwell method to treat the

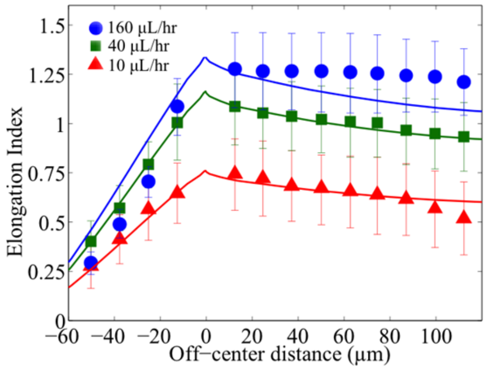

FIGURE 1 Comparison of experimental results[7] and simulations. There is good agreement, especially at P = 10 μL/hr and P = 40 μL/hr.

viscoelastic terms of the Navier—Stokes equations,

$$\tau + \lambda\frac{\partial}{\partial t}\tau + \lambda v \cdot \nabla\tau - \lambda(\nabla v)^T \cdot \tau - \lambda\tau \cdot \nabla v = 2\eta_0 D,$$

where **τ** is the stress tensor, λ the relaxation time, **v** the fluid viscosity, **D** the tensor of the deformation rate and η_0 the viscosity at steady shear. The simulations show good agreement with the experimental results, especially at lower flow rates. At higher flow rates, nonlinear effects on the cell are more important and the agreement is somewhat inferior. The Elongation Index is defines as $EI = \frac{L - \bar{L}}{\bar{L}}$, where L is the length of the major axis and $\bar{L}$ is the length of the mean undeformed major axis.

References

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