Supplementary Information

Polarization-sensitive color in the iridescent scales of *Ornithoptera* butterfly

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Part A

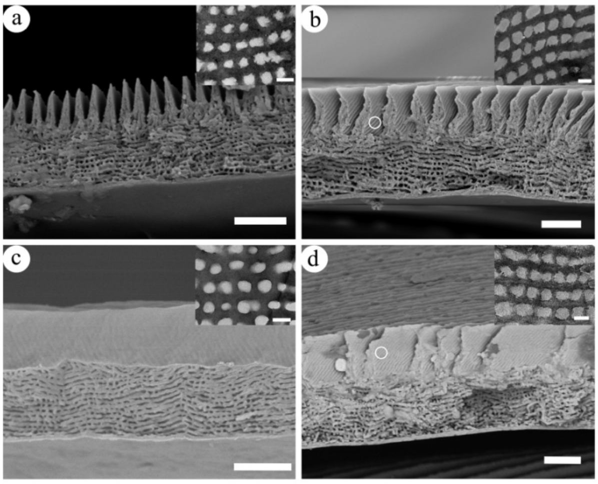

Fig. S1 FESEM and TEM images of *O.p.poseidon* green scales and *O.c.lydius* orange scales (a) Cross-section of green scale. The inset shows the matching multilayer structure. (b) Cross-section of orange scale. The inset shows the matching multilayer structure. (c) Longitudinal-section of green scale. The inset shows the multilayer structure. (d) Longitudinal-section of orange scale. The inset shows the matching multilayer structure. (a - d) Scales bar: 2μm; (Inset Scales bar: 200 nm). (The cross-section is perpendicular to the longitudinal axis of ridges, and the longitudinal-section is parallel to the longitudinal axis of ridges).

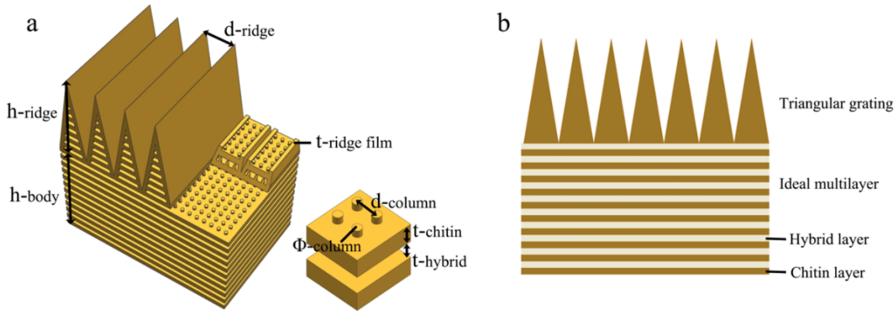

Fig. S2 Schematic model of green and orange scales and the simplified combined architecture. (a) Schematic model of green and orange scales. h_{ridge} represents the height of ridges, d_{ridge} represents the distance of adjacent ridges, h_{body} represents the height of body multilayer, $t_{ridge\ wall}$ represents the thickness of the ridge wall, Φ_{column} represents the diameter bracing column, d_{column} represents the distance of adjacent bracing columns, t_{hybrid} represents the thickness of the hybrid layer in multilayer, t_{chitin} represents the thickness of the chitin layer in multilayer. (b) The simplified combined architecture. The combined architecture is comprised of upper triangular grating and bottom multilayer.

Table S1 Dimensions of anatomy structures for green scales and orange scales

Butterflies	h-ridge /μm	d-ridge /μm	h-body /μm	t-chitin /μm	t-hybrid /μm	t-ridge wall /μm	Φ-column /μm	d-column /μm
O.p.poseidon	1.90	0.54	2.50	0.10	0.11	0.13	0.06	0.16
O.c.lydius	2.60	0.76	2.60	0.12	0.14	0.14	0.05	0.18

h_{ridge} represents the height of ridges, d_{ridge} represents the distance of adjacent ridges, h_{body} represents the height of body multilayer, $t_{ridge\ wall}$ represents the thickness of the ridge wall, Φ_{column} represents the diameter bracing column, d_{column} represents the distance of adjacent bracing columns, t_{hybrid} represents the thickness of the hybrid layer in multilayer, t_{chitin} represents the thickness of the chitin layer in multilayer.

Part B

Text S1 Calculation of the transmitted ±*1st* order diffraction angle and the reflected wavelength

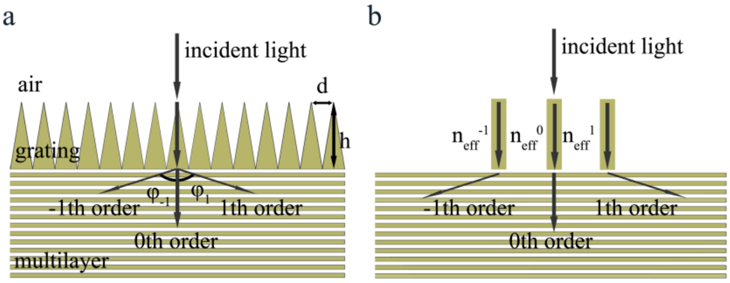

Fig. S3 Schematic diagram of modulated structure at normally incident light and the modal method model. (a) Diffraction behavior of grating in modulated structure. Incident light normally passes through the grating region, and the *0th* and *±1st* order diffraction occur. (b) Modal method model for the diffraction behavior of deep grating. The propagation of different order lights through the grating region is similar to the slab waveguide with discrete modes.

Step1:

The diffraction angle of the transmitted diffraction lights can be calculated by the grating equation [2]:

$$n_{out}\sin\theta_m = n_{in}\sin\theta_{in} + \frac{m\lambda}{d} \qquad \text{[Eq.S1.1]}$$

where n_{in} is the refractive index of the air ($n_{air}=1$); n_{out} is the refractive index of the chitin (n_c =1.56); θ_{in} and θ_m are the angle of incidence and the angle of the m th diffractive order, respectively; λ is the incident wavelength, d is the grating period, m is the diffraction order. At normal incidence, the angle of incidence is zero. The diffraction angles of the transmitted $\pm 1st$ diffraction orders are given by, $\theta_{\pm 1} = \frac{\pm\lambda}{n_c d}$, (Fig. S3). (n_c is the refractive index of the chitin)

step2:

The transmitted $\pm 1st$ diffraction orders lights illuminate on the lower multilayer with their respective diffraction angles, and causing multilayer interference. The constructive interference equation of multilayer is given by [3],

$$n_c t_c \cos\theta_c + n_h t_h \cos\theta_h = \frac{m\lambda}{2} \qquad \text{[Eq.S1.2]}$$

where n_c (n_h), t_c (t_h), and θ_c (θ_h) are the RI, the thickness, and the refractive angle of the chitin layer and hybrid layer, and λ is incident light wavelength and *m* is integer. n_c and n_{air} are 1.56 and 1.00, respectively. n_h is 1.08 for green scales and 1.04 for orange scales.

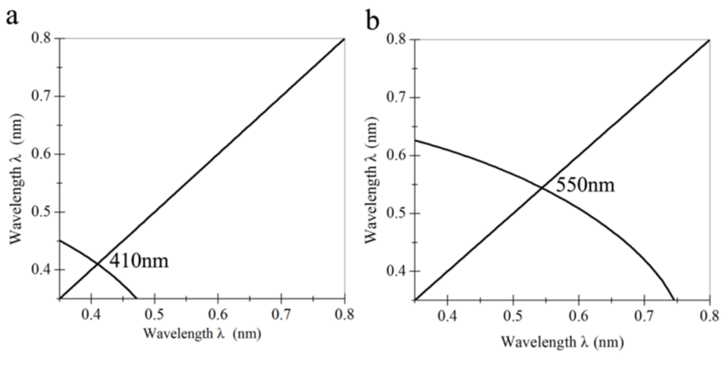

Fig. S4 Reflection peaks of the *±1st* mode lights for green and orange scales. (a) The reflection peak of the *±1st* mode lights for green scales is about 410nm. (b) The reflection peak of the *±1st* mode lights for orange scales is about 550nm.

Step3:

Thus, the light which satisfies the grating equation and the constructive interference equation of multilayer can be reflected back. the wavelength of the reflected $\pm 1st$ mode lights is calculated by the simultaneous equation

$$\begin{cases} \theta_{\pm 1} = \dfrac{\pm\lambda}{n_c d} \\ n_c t_c \cos\theta_c + n_h t_h \cos\theta_h = \dfrac{m\lambda}{2} \end{cases} \qquad \text{[Eq.S1.3]}$$

where θ_h is given by $n_h = \sin\theta_c n_c / \sin\theta_h$

For the green scales (d=0.52 um), the reflected $\pm 1st$ mode lights are at about 410 nm wavelength. For orange scales (d=0.76 um), the reflected $\pm 1st$ mode lights are at about 550 nm wavelength (Fig. S4).

Part C

TextS2 Calculation of the phase difference δ_i for form-birefringence:

Step1:
For the birefringence wafer, whose thickness is h , when polarized light that passes through the phase difference, δ is given by [5],

$$\delta = \frac{2\pi}{\lambda}(n_{\parallel} - n_{\perp})h \qquad \text{[Eq.S2.1]}$$

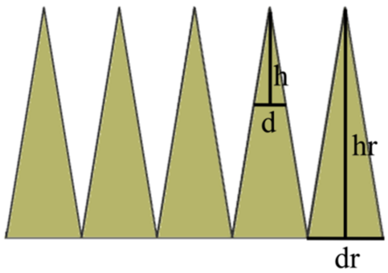

Fig. S5 The height profile of the effective refractive index value for calculation. h is the distance to the top of the ridge, and h_r is the height of the ridge. f is the volume fraction of ridges at the matching horizontal plane. $(f = \frac{d}{d_r} = \mathrm{h}/\mathrm{h}_r)$

where $n_{\parallel}$ and $n_{\perp}$ are the refractive indices in the parallel optics axis orientation and perpendicular to the optics axis orientation.
Step2:
For the gradual width from top to bottom, the phase difference of the decomposed light caused by the grating is calculated with the integration method. Where d_h is infinitesimal segment, h is its distance to the top of the ridge, and h_r is the height of the ridge. f is the volume fraction of ridges at the matching horizontal plane (Fig. S5). f is given by

$$f = h / h_r \qquad \text{[Eq.S2.2]}$$

and the form-birefringence effective medium theory [4] is given by

$$\begin{cases} n_{\parallel}^2 = f n_r^2 + (1-f) n_{air}^2 \\ 1/n_{\perp}^2 = f / n_r^2 + (1-f)/n_{air}^2 \end{cases} \qquad \text{[Eq.S2.3]}$$

where $n_{\parallel}$ and $n_{\perp}$ are the effective RIs in the parallel grating orientation and the perpendicular grating orientation. Thus, we define $d_{\delta i1}$ as the phase difference of infinitesimal segment. $d_{\delta i1}$ is given

$$d_{\delta i1} = \frac{2\pi}{\lambda}(n_{\parallel} - n_{\perp}) d_h \qquad \text{[Eq.S2.4]}$$

The phase difference of tapered grating is given by,

$$\delta_{i1} = \int_0^{h_r} \frac{2\pi}{\lambda}(n_{\parallel} - n_{\perp}) d_h \qquad \text{[Eq.S2.5]}$$

We define δ_{i2} as the phase difference of the light reflected by multilayer. P-polarized optical vector is the same with the s-polarized optical vector regarding multilayer interference at normal incident light. Thus,

$$\delta_{i2} = 0 \qquad \text{[Eq.S2.6]}$$

Then, the reflected polarized light transmits the upper tapered grating. We define δ_{i3} as the phase difference of this process. δ_{i3} is given by

$$\delta_{i3} = \delta_{i1} \qquad \text{[Eq.S2.7]}$$

So, the phase difference δ_i of the light reflected by the modulated architecture is given by

$$\delta_i = 2\int_0^{h_r} \frac{2\pi}{\lambda}(n_{\parallel} - n_{\perp}) d_h \qquad \text{[Eq.S2.8]}$$

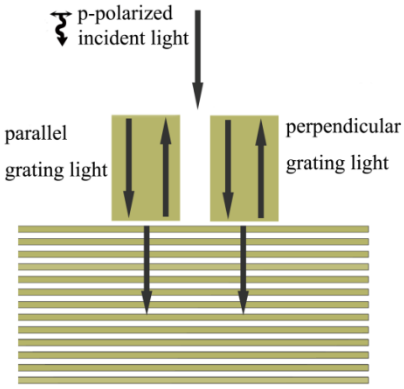

Fig. S6 Schematic diagram of the incident p-polarized light pass through grating structure. Incident p-polarized light is decomposed into parallel grating polarized light and perpendicular grating polarized light.

TextS3 Calculation of the intensity for reflected polarized light:

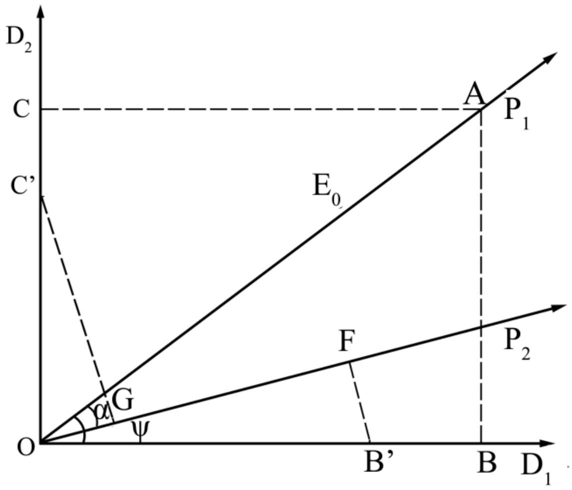

Fig. S6 Schematic diagram of the derivation process of the intensity for reflected polarized light. Form-birefringence sample under polarization microscope. D_1 is the optics axis of form-birefringence, and D_2 is perpendicular to the optics axis of form-birefringence. P_1 is the orientation of the transmitted axis of the input polarizer, and P_2 is the orientation of the transmitted axis of the output analyzer. The included angle of P_1 and P_2 is α, and the included angle of P_1 and D_1 is φ. Incident light passes through the input polarizer, illuminating on the form-birefringence model, and the amplitude of the achieved polarized light is *OA*. We decomposed *OA* into two parts, *OB* and *OC*. *OB* is the amplitude of the component polarized light along the axis of the form-birefringence model, and *OC* is the amplitude of the component polarized light perpendicular to the axis of the form-birefringence model.

Step1:

For the interference intensity I of two same frequency waves in the same the direction is given by $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$, whose phase difference is δ [5].

Step2:

Here we calculate the interference intensity of two decomposed lights to obtain the intensity of the reflected polarized light. Under the polarization microscope, the intensity of the incident p-polarized light is I_{i0}, the wavelength of incident light is λ_i. R_i is the reflectance of the bottom multilayer, and δ_i is the phase difference caused by upper grating.

As shown in the Fig. S6, D_1 is the optics axis of form-birefringence (upper grating), and D_2 is perpendicular to the optics axis of form-birefringence (upper grating). P_1 is the orientation of the transmitted axis of the input polarizer, and P_2 is the orientation of the transmitted axis of the output analyzer. The included angle of P_1 and P_2 is α, and the included angle of P_1 and D_1 is φ. The amplitude of the incident p-polarized light is *OA*. We decomposed *OA* into two parts, *OB* and *OC*. *OB* is amplitude of component polarized light along D_1 and *OC* is the amplitude of the component polarized light along D_2. $OA = E_{io}$, and $I_{i0} = kE_{i0}^2$ (k is constant)

$$\begin{cases} OB = E_{i0}\cos\varphi \\ OC = E_{i0}\sin\varphi \end{cases} \quad \text{[Eq.S3.1]}$$

The bottom multilayer selectively reflects the incident light. OB and OC are the amplitudes of the decomposed incident lights. OB' and OC' are the amplitudes of the matching reflected polarized light of OB and OC, respectively.

$$\begin{cases} OB' = E_{i0}\sqrt{R}\cos\varphi \\ OC' = E_{i0}\sqrt{R}\sin\varphi \end{cases} \quad \text{[Eq.S3.2]}$$

OF is the component amplitude of polarized light OB' that passes through the output analyzer, and OG is the component amplitude of polarized light OC' that passes through the output analyzer.

$$\begin{cases} OF = E_{i0}\sqrt{R}\cos\varphi\cos(\varphi - \alpha) \\ OG = E_{i0}\sqrt{R}\sin\varphi\sin(\varphi - \alpha) \end{cases} \quad \text{[Eq.S3.3]}$$

Thus, the reflected polarized amplitude is E_i ,

$$E_i^2 = E_{i0}^2 R[\cos^2\varphi\cos^2(\varphi - \alpha) + \sin^2\varphi\sin^2(\varphi - \alpha) + 2\cos\varphi\cos(\varphi - \alpha)\sin\varphi\sin(\varphi - \alpha)\cos\delta_i] \quad \text{[Eq.S3.4]}$$

Thus,

$$I_i = I_{i0}R_i[\cos^2\alpha - \sin 2\varphi in2(\varphi - \alpha)\sin^2(\delta_i/2)] \quad \text{[Eq.S3.5]}$$

For the crossed polarizers, $\alpha = 90°$, and $\varphi = 45°$, and the phase difference is δ_i, the s-polarized reflection intensity $I_{iR}(s)$ under the crossed analyzer and the p-polarized reflection intensity $I_{iR}(p)$ under the collinear analyzer are given by

$$\begin{cases} I_{iR}(s) = I_{i0}\, R_i \sin^2(\delta_i/2) \\ I_{iR}(p) = I_{i0}\, R_i \cos^2(\delta_i/2) \end{cases} \quad \text{[Eq.S3.6]}$$

References

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