Electronic Supplementary Information (ESI)

# Mechanical properties of normal and binormal double nanohelices

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## I. The mathematical expression of the double helix

According to the Cosserat curve model (see ref. 33), the mathematical expression of the double helix $H_{DI}$ is

$$\begin{aligned}
\boldsymbol{x} = &\left[\left(\frac{\boldsymbol{F}'}{\boldsymbol{E}_1}\sin\theta+\frac{\boldsymbol{f}}{\hat{\psi}}\frac{1}{\boldsymbol{E}_1}\cos\theta\right)(-\cos\theta)+\left(\frac{\boldsymbol{F}'}{\boldsymbol{E}_3}\cos\theta-\frac{\boldsymbol{f}}{\hat{\psi}}\frac{1}{\boldsymbol{E}_3}\sin\theta+1\right)\sin\theta\right]\cos\psi\boldsymbol{e}_1 \\
&+\left[\left(\frac{\boldsymbol{F}'}{\boldsymbol{E}_1}\sin\theta+\frac{\boldsymbol{f}}{\hat{\psi}}\frac{1}{\boldsymbol{E}_1}\cos\theta\right)(-\cos\theta)+\left(\frac{\boldsymbol{F}'}{\boldsymbol{E}_3}\cos\theta-\frac{\boldsymbol{f}}{\hat{\psi}}\frac{1}{\boldsymbol{E}_3}\sin\theta+1\right)\sin\theta\right]\sin\psi\boldsymbol{e}_2\,, \qquad \text{(S1)} \\
&+\left[\left(\frac{\boldsymbol{F}'}{\boldsymbol{E}_1}\sin\theta+\frac{\boldsymbol{f}}{\hat{\psi}}\frac{1}{\boldsymbol{E}_1}\cos\theta\right)\sin\theta+\left(\frac{\boldsymbol{F}'}{\boldsymbol{E}_3}\cos\theta-\frac{\boldsymbol{f}}{\hat{\psi}}\frac{1}{\boldsymbol{E}_3}\sin\theta+1\right)\cos\theta\right]\boldsymbol{e}_3
\end{aligned}$$

where $\boldsymbol{e}_i$ (i=1,2,3) are the orthogonal basis of the fixed rectangular Cartesian system.

## II. Interlocking helix angle

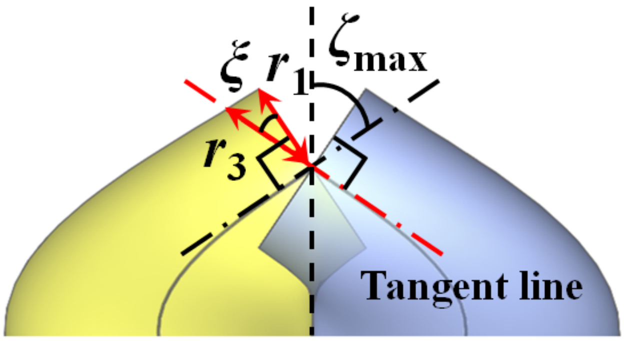


Figure S1. A section of a tightly packed normal double helix.

For the normal and binormal twisted double nanohelices with elliptic cross sections, $\zeta_{max}$ is determined by the semiminor axis of $r_1$and semimajor axis of $r_2$. Fig. 2(a) shows a section of tightly packed normal double helix. The black line is the centre line of the rod and $T$ is a contacting point on the helix axis. We have

$$\rho = \rho_T + r_2\,, \qquad \text{(S2)}$$

where $\rho$ is the curvature radius of the rod centre line, *i.e.* the curvature radius of the double helix, and $\rho_{\mathrm{T}}$ is the curvature radius along the tangent line direction of the blue rod centre line at the contacting point *T*. The schematics on the left panel in Fig. 2(b) displays a tightly packed normal double helix with the number of turns 1/4. The black and red dash-dotted lines are along the tangent line direction of the yellow and blue rod centre line at the points of the corresponding contacting point *T*, respectively. The schematics on the right panel in Fig. 2(b) presents the elliptic cross section of the yellow rod with semi-axes $r_2$, $r_3$ along the tangent line direction of the blue rod centre line, *i.e.* the red dash-dotted line.

As displayed in Figure S1, we mark the angle of $\xi$ in Fig. 2(b) to explain the semi-axis $r_3$ in detail. The relationship of

$$r_3 = \frac{r_1}{\cos\xi} \tag{S3}$$

can be obtained. Since $\xi = 2\zeta_{\max} - \pi/2$, we have

$$r_3 = \frac{r_1}{\cos(2\zeta_{\max} - \pi/2)}. \tag{S4}$$

When a normal double nanohelix is tightly packed, $\rho_{\mathrm{T}}$ is equal to the curvature radius of the yellow elliptic cross section of the rod at the contacting point *T*, shown in Supplementary-Figure1, which leads to

$$\rho_{\mathrm{T}} = \frac{r_3^2}{r_2}. \tag{S5}$$

Combining Eqs. (S2), (S4) and (S5) with the curvature radius of the double helix $\rho = \frac{4\pi^2 r_2^2 + b_0^2}{4\pi^2 r_2}$, we can obtain

$$b_0 = \frac{2\pi r_1}{\cos(2\zeta_{\max} - \pi/2)}. \tag{S6}$$

Substituting Eq. (S6) in the relationship of $\tan\zeta_{\max} = \frac{2\pi r_2}{b_0}$, we have

$$\zeta_{\max} = \arccos\sqrt{\frac{r_1}{2r_2}} \tag{S7}$$

for a normal double helix. Similarly,

$$\zeta_{\max} = \arccos\sqrt{\frac{r_2}{2r_1}} \tag{S8}$$

for a binormal double helix can be derived.