The role of multiple representations in the understanding of ideal gas problems

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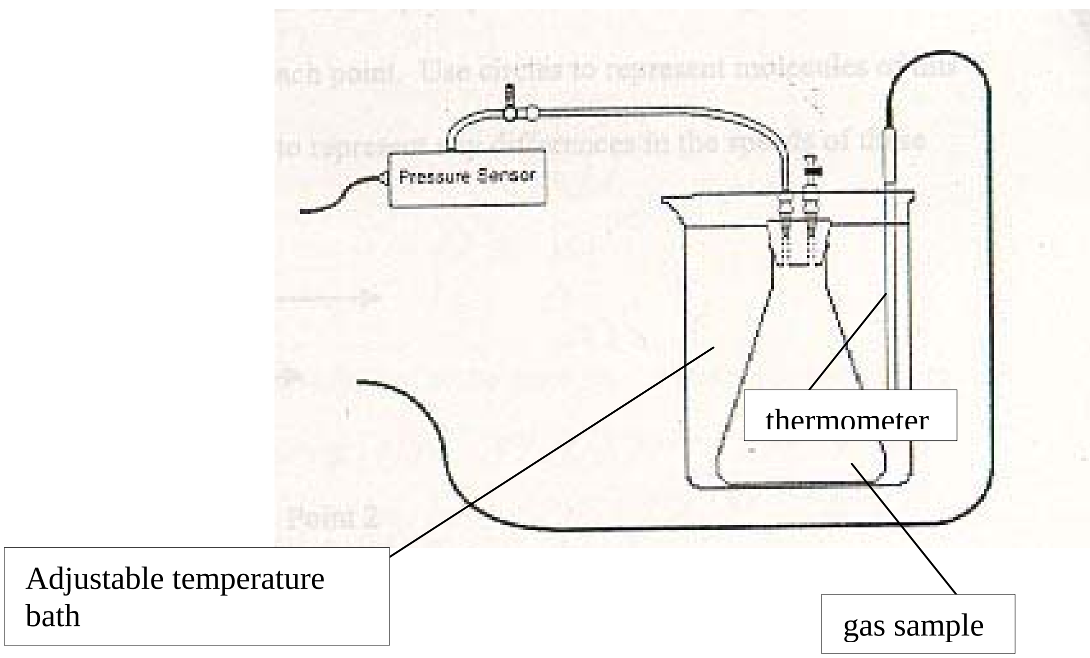
Supplementary Data

SECTION 1. Verbal Protocol for Think-Aloud Volunteers

Part I—Problem Solving Task

Here is a picture of an apparatus that can be used to measure the pressure of a gas sample at various temperatures (Figure used with permission of Vernier):



Assume that this device could be used to take pressure readings on gas samples for a range of gas temperatures between 25.0 °C and 100.0 °C. Assume gas samples behave ideally.
Question 1: What will happen if the gas is heated? Please explain your answer.
Answer. The measured pressure will increase as the gas is heated.

Question 2: If the flask contains 0.1 moles of an ideal gas, the volume of the flask is 1L, the gas constant is $R = 8.206 \times 10^{-2}$ (L·atm)/(mol·K), and the initial temperature is 25 °C, what is the initial pressure of the gas?
Answer. approximately 2.44 atm.

Question 3: If this same sample of gas is heated to 100 °C, what will the final pressure be?
Answer. 3.06 atm (approximately).

Part II--Think Aloud Session

Question 4: If you could graph the data from this experiment, what do you think this graph would look like? Please sketch your graph in the space below.
Answer. Temperature should be plotted on the horizontal axis and pressure on the vertical axis with a linear trend line representing the relationship.

Question 5: If you could see the motion of the gas molecules at 0 °C and at 100 °C, what would they look like in comparison? Please draw your sketch below.
Answer. Students were expected to draw gas molecules as spheres or circles impacting the walls of the container; gas molecules at the higher temperature move with greater average velocity thereby creating greater pressure. The number of molecules in both cases should be shown as equal and the motion of the molecules should be random.

The following pages contain graphs, molecular sketches, algebraic equations, and data tables. On a scale of 1 to 4 (1 = the most valuable and 4 = the least valuable) rank these graphs, molecular sketches, equations, and data tables for their usefulness to you in understanding the problem.

	Most			Least
____Graphs	1	2	3	4
____Molecular sketches	1	2	3	4
____Equations	1	2	3	4
____Data tables	1	2	3	4

PV = nRT

Caption. The ideal gas law equation. The ideal gas law contains all the relations describing the response of an ideal gas sample to changes in pressure, volume, temperature, and amount of molecules. That is, pressure and volume are inversely proportional to one another. Pressure is directly proportional to amount and temperature. Volume is directly proportional to amount and temperature. For pressure in atm and volume in L, $R = 8.206 \times 10^{-2}$ L·atm·K^{-1}·mol^{-1}.

Temperature (^{o}C)	Pressure (atm)
0	2.24
10	2.34
20	2.40
30	2.49
40	2.57
50	2.65
60	2.73
70	2.81
80	2.90
90	2.98
100	3.06

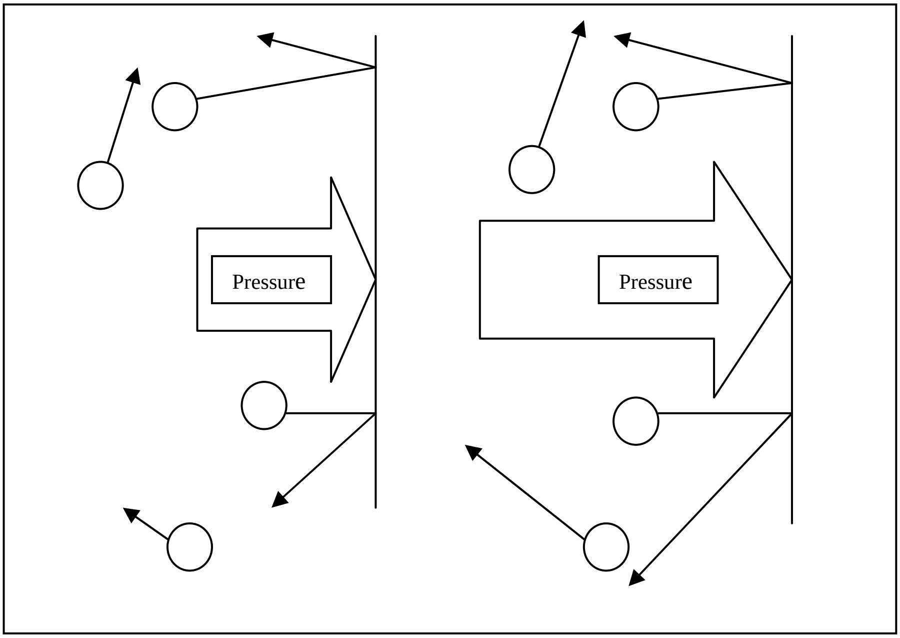


Figure Caption.

a. As we have seen, the pressure of a gas sample arises from the impact of its molecules with the container walls.

b. When the temperature of the sample is increased the molecules are moving with greater velocity, so there will be more collisions with walls in a given time than at a lower temperature. Because the impact on the walls is greater now, so is the gas pressure. In other words, increasing the temperature of a gas sample at constant volume increases its pressure.

Note: This representation of the molecular behavior of gas molecules is a possible projection of the three-dimensional flight of gas particles in a container and reflecting off the wall of the container. The arrows can be thought of as velocity vectors. The figure was presented to student volunteers as a two-dimensional projection of a three-dimensional phenomenon. It is a modification of a textbook figure representing the effect of volume changes on the pressure of a gas (Jones and Atkins, 2000, p. 184).

Section 2. Representational Competence Think Aloud Coding Rubric[1]

Level 1: Representations as depictions

Component 1, Using representations to explain physical phenomena

1.1.1. When asked to represent a physical phenomenon, the person generates representations of the phenomenon based only on its physical features. That is, the representation is an isomorphic, iconic depiction of the phenomenon at a point in time.

Component 2, Interpreting or explaining the meaning of a representation

1.2.1. The person is not familiar with a provided, formal representational system or does not know what it is use for. They are not able to identify or analyze the features or patterns of features for a particular representation.

Level 2: Early Symbolic Skills

Component 1, Using representations to explain physical phenomena

2.1.1. When asked to represent a physical phenomenon, the person generates representations of the phenomenon based on its physical features, but also includes some symbolic elements to accommodate the limitations of the medium (e.g., use of symbolic elements such as arrows to represent dynamic notions, such as time or motion or an observable cause, in a static medium, such as paper). However, non-perceptual or non-observed aspects of the phenomenon (e.g., molecular entities or processes) are not represented.

[1] Highlighted codes were not used for scoring the transcripts of this study. Only observable behaviors were recorded and coded.

Component 2, Interpreting or explaining the meaning of a representation

2.2.1. The person is familiar with a provided, formal representational system and what it is used for, but an explanation of a representation is merely a literal reading of its surface features without regard to syntax and semantics.

2.2.2. The person does not make reference to the phenomenon that the representation stands for.

Component 3, Using representations together

2.3.1. The person is not able to make connections between multiple representations of the same phenomenon.

2.3.2. The person transforms a given representation with significant errors.

Level 3: Syntactic Use of Formal Representations

Component 1, Using representations to explain physical phenomena

3.1.1. When asked to represent a physical phenomenon, the person generates representations of the phenomenon based on both observed physical features and unobserved, underlying entities or processes (such as an unobserved cause). The representational system may be invented and idiosyncratic and the represented entities or processes may not be scientifically accurate or violate scientific convention.

Component 2, Interpreting or explaining the meaning of a representation

3.2.1. The person is able to use words to correctly identify and analyze the features and patterns of features of provided formal representations based on syntactic rules only. The person is able to verify the results of a calculation.

3.2.2. The person makes a connection between a formal representation and the physical phenomenon that it stands for.

3.2.3. However, the person is not able to use the features of a provided representation to support a prediction or claim about the phenomenon.

Component 3, Using representations together

3.3.1. The person is able to correctly make connections across two different representations and explain the relationship between them, but this relationship is based only on syntactic rules or shared surface features, rather than the shared, underlying meaning of the different representations and their features.

3.3.2. Given a representation, the person generates a representation of the same phenomenon in another symbol system with idiosyncrasies or scientific inaccuracies.

Component 4, Social use of representations

3.4.1. The person does not spontaneously use representations to explain a phenomenon in a social situation.

Component 5, Reflective use of representations

3.5.1. The person does not display a notion that the representation is in some sense independent from its referent and can be evaluated as a good or bad representation based on some criterion.

Component 6, Epistemological position of representations

3.6.1. The person holds or displays the epistemological position that the representation is in some sense the scientific phenomenon being studied or confuses the difference between the scientific phenomenon and its representation.

Level 4: Semantic, Social Use of Formal Representations

Component 1, Using representations to explain physical phenomena

4.1.1. When asked to represent a physical phenomenon, the person spontaneously and correctly uses a formal symbol system to represent underlying, non-observable entities and processes.

4.1.2. The person is able to use the features of a representation to support a prediction or claim about the phenomenon or to make a prediction or formulate a hypothesis about the phenomenon at some future date or under some different condition.

Component 2, Interpreting or explaining the meaning of a representation

4.2.1. The person is able to use words to identify and analyze the features and patterns of features of a provided, formal representation based on both syntactic rules and meaning, relative to some physical phenomenon that it represents. He or she is able to use the representation to explain the phenomenon beyond its observable physical features.

Component 3, Using representations together

4.3.1. The person is able to make connections across two different representations or transforms one representation to another. The person is able to explain this relationship based on the shared meaning of the different representations and their features.

4.3.2. The person transforms a given representation into an equivalent representation in another symbol system, but this transformation contains minor violations of scientific convention.

Component 4, Social use of representations

4.4.1. The person spontaneously uses representations to explain a phenomenon, support a claim, or make a prediction in a social situation.

Component 5, Reflective use of representations

4.5.1. The person is able to select or construct an appropriate representation to use in a particular situation and is able to explain why that representation is appropriate but does not or cannot compare the appropriateness of its use relative to other representations.

Component 6, Epistemological position of representations

4.6.1. The person is able to take the epistemological position that the representation corresponds to, but is distinct from the phenomenon it represents.

Level 5: Reflective, Rhetorical Use of Representations

Component 1, Using representations to explain physical phenomena

5.1.1. When asked to explain a physical phenomenon, the person uses one or more representations to explain the relationship between physical properties and underlying entities and processes.

Component 2, Interpreting or explaining the meaning of a representation

5.2.1. A person generates one representation to explain or understand the meaning of another.

5.2.2. A person recognizes mistakes or inadequacies in a provided representation.

Component 3, Using representations together

5.3.1. The person can appropriately provide a common underlying meaning for a number of different kinds of superficially different representations.

5.3.2. The person is able to transform any given representation into an equivalent representation in anther form.

5.3.3. The person uses the features or patterns of features of one representation to validate or confirm inferences drawn from another representation.

5.3.4. The person transforms a given representation into an equivalent representation in another symbol system without errors and according to scientific convention.

Component 4, Social use of representations

5.4.1. The person is able to use features of one or more representations to construct an argument contrary to the position of another person.

Component 5, Reflective use of representations

5.5.1. The person is able to comment on the quality of the representation based on its syntax or on its semantics in relation to the phenomenon it represents.

5.5.2. The person makes distinctions between data-generated and person-generated representations and provides reasons for the authority of one over the other.

5.5.3. The person is able to select or construct an appropriate representation to use in a particular situation and explain why that representation is more appropriate than another is.

5.5.4. The person is able to use a formal representation in an idiosyncratic or abbreviated way that emphasizes important elements and de-emphasizes unimportant ones when explaining some phenomenon to an intended audience.

5.5.5. If the person is not able to find an appropriate, established representation to explain an idea or phenomenon, he or she invents one and uses it effectively to communicate the idea.

Component 6, Epistemological position of representations

5.6.1. The person is able to take the epistemological position that we are not able to directly experience certain phenomena and these can be understood only through their representations. Consequently, this understanding is open to interpretation. Confidence in an interpretation is increased to the extent that representations can be made to correspond to each other in important ways, particularly correspondence between data-generated and human generated representations.

Level 6: Affective Response to Representations

Components to be developed according to evidence in the transcripts.

Level 7: Clarifications or confirmations; statements not directly related to representational competence.

Highlighted items correspond to codes that the evaluators deemed inappropriate. These codes were to be assigned to non-observed behaviors. These codes would have required the evaluators to speculate about the subjects' state of mind in the absence of evidence. The evaluators preferred to judge the subjects according to observable, recorded behavior, which nevertheless required an element of subjective judgment.

Section 3. Excerpts from Verbal Protocol Transcripts

Below are two excerpts from the think aloud protocols. In each case students are asked the same questions in the same manner. The first student volunteer, with the pseudonym Noble, was one of two upper level students. The second volunteer, designated Copper, was a lower level or beginning student. The first column identifies the speaker, the second column is a transcript of the dialogue, and the third column represents how the researchers coded the responses according the rubric. A count of the coded responses from these and the remaining transcripts led to the summary data presented in Tables 3 and 4 in the text.

Speaker	**Dialogue**	**Coding**
NOBLE's Interview		
	Question 1: What will happen if the gas is heated? Please explain your answer.	
SM	Look at the pictures. We have pressures, a recording device receiving input from pressure sensor and thermometer. We have both pressure and temperature and we have	

	fixed volume. (Reads the question again).	
NOBLE	On the molecular level?	4.1.1, 4.3.1, 7, Noble is clarifying the question but also revealing a deeper understanding.
SM	You could do that.	
NOBLE	The first thing I thought of was seeing little molecules going quicker.	4.1.1, 4.3.1, She is already going beyond the device diagram to the molecular level.
SM	Okay. You can describe that for me orally; why don't you do that first, rather than feeling like you need to write it down on paper. Tell me what you're thinking rather than trying to write it on paper.	
NOBLE	The way I think of it is when they're at a lower temperature, they're all just kind of hanging out at the bottom of the thing. They're bouncing around, but once they start getting heated up, they start moving around a little bit more and a lot quicker, so the more temperature that's added then they start going faster and faster.	3.1.1, 3.3.2, An apparent misconception, molecular behavior and temperature. 3.1.1, 3.3.2, Apparently she has already corrected herself.
SM	They being?	
NOBLE	Molecules.	7
	*Question 2: If the flask contains 0.1 moles of an ideal gas, the volume of the flask is 1L, the gas constant is $R = 8.206 \times 10^{-2}$ L*atm/moles*K, and the initial temperature is 25°C, what is the initial pressure of the gas?*	
NOBLE	We have to rearrange our equation. We have $PV = nRT$. . .I need more space. (Writing out the equation) Oh, we have to convert to Kelvin. All of that divided by the volume, which is one liter. . .all right. (Gets calculator). The pressure equals 2.45 atmospheres.	4.1.1, She immediately recognizes the need for the equation and rearranges it as necessary.
COPPER's Interview		
	Question 1: What will happen if the gas is heated? Please explain your answer.	
SM	Look at the pictures. We have pressures, a	

	recording device receiving input from pressure sensor and thermometer. We have both pressure and temperature and we have fixed volume.	
COPPER	If the gas is heated, the pressure, wait—you mean overall what's going to happen?	7, Clarification
SM	Right, just a very general question. What's going to happen if you heat that up?	
COPPER	If you heat it up, then the pressure—hold on a second . . .okay, if you heat this up, then the pressure is going to be increased.	4.1.1
SM	Okay.	
COPPER	Because. . . .	7, Thinking
SM	What are you thinking?	
COPPER	I'm thinking, I'm not sure if. . . I'm reading the question.	7, Thinking
SM	Oh, okay.	
COPPER	That since you're heating it up, the particles will move faster, which will increase the pressure, but I'm not sure.	4.1.1, 4.2.1, 4.3.1, 6
SM	Okay.	
COPPER	Can I go onto the next question?	6,7, Perhaps an element of self-doubt or impatience?
SM	Absolutely. Go on to question #2.	
	*Question 2: If the flask contains 0.1 moles of an ideal gas, the volume of the flask is 1L, the gas constant is $R = 8.206 \times 10^{-2}$ L*atm/moles*K, and the initial temperature is 25°C, what is the initial pressure of the gas?*	
COPPER	So. . . .I'm thinking. . .	7, Thinking
SM	Okay. Tell me what you're thinking if you can while you're doing that.	

COPPER	Okay, I'm just trying to think of an equation that goes along with this, which is. . .(reads it over again and starts writing).	7, She seems to be thinking of an equation as an algorithm.
SM	What are you writing?	
COPPER	The equation *PV* = *mRT*—well, it's supposed to be *nRT*, but I never use the *n*.	3.1.1, 4.1.1, She identifies the correct equation, but she says she uses m (probably because it is the first letter of moles) rather than n.
SM	Okay.	
COPPER	Okay, but wait now. You have to have—okay, yeah.	7, Thinking
SM	What were you thinking?	
COPPER	I was thinking that you don't have a pressure, but that's what we're trying to figure out.	3.2.1
SM	Okay.	
COPPER	You take the pressure times the volume equals the mass. . . (writing the equation).	3.1.1
SM	What are you doing right now?	
COPPER	Writing out the equation.	7
SM	Okay.	
COPPER	And the temperature is 25° C.	7
SM	What are you doing now?	
COPPER	Changing to it Kelvin.	3.2.1
SM	Okay.	
COPPER	(Writing and using calculator) Oh man. . .	6, She's getting into it.
SM	What are you trying to do?	
COPPER	Add the—do I have to use this one? Can I use my own calculator?	6,7, Clarification. She prefers her own scientific calculator.
SM	Oh yes, use your own calculator; by all means, please do. I just thought maybe the battery	

	went out all of a sudden.	
COPPER	Mine's just more little.	6, calculator preference
SM	Yes, use whatever you're comfortable with; you bet.	
COPPER	This is hard.	6, Referring to the graphing calculator provided.
SM	We'll talk about it afterwards.	
COPPER	Okay.	7, Clarification
SM	What are you doing now?	
COPPER	Canceling the units.	3.2.1
SM	Okay.	
COPPER	And then I'm going to. . . (talking to herself and figuring). Okay, 2.445.	3.2.1, See written work, she obtains the correct answer, but apparently writes it incorrectly.