## Electronic Supplementary Information

# How to Determine Accurate Chemical Ordering in Several Nanometer Large Bimetallic Crystallites from Electronic Structure Calculations 

Sergey M. Kozlov, ${ }^{\text {a }}$ Gábor Kovács, ${ }^{\text {a }}$ Riccardo Ferrando, ${ }^{\text {b }}$ Konstantin M. Neyman ${ }^{\mathrm{a}, \mathrm{c}^{*}}$<br>${ }^{\text {a }}$ Departament de Química Física and Institut de Química Teòrica i Computacional (IQTCUB), Universitat de Barcelona, c/ Martí i Franquès 1, 08028 Barcelona, Spain<br>${ }^{\text {b }}$ Dipartimento di Fisica and CNR-IMEM, Via Dodecaneso 33, 16146 Genova, Italy<br>${ }^{\text {c }}$ Institució Catalana de Recerca i Estudis Avançats (ICREA), 08010 Barcelona, Spain<br>* E-mail: konstantin.neyman@icrea.cat

## 1. Relation between topological energy expression and model Hamiltonians

For nanoparticles (NPs) with a fcc lattice and extended $\{111\}$ facets exposed, there is a relation between the topological energy expression

$$
\begin{equation*}
E_{T O P}=E_{0}+\varepsilon_{B O N D}^{A-B} N_{B O N D}^{A-B}+\varepsilon_{C O R N E R}^{A} N_{C O R N E R}^{A}+\varepsilon_{E D G E}^{A} N_{E D G E}^{A}+\varepsilon_{T E R R A C E}^{A} N_{T E R R A C E}^{A} \tag{1}
\end{equation*}
$$

and a model Hamiltonian $\widehat{H}_{T O P}$ acting on local degrees of freedom of the system. Namely, let $\hat{\sigma}_{i}$ be a fictitious atomic spin operator that upon acting on the wave function of the system returns value 1 , if the lattice site $i$ is occupied by atom $A$ and -1 , if the site $i$ is occupied by atom $B$. Then, one can express the number of heteroatomic bonds $A-B$ in the system as

$$
\begin{equation*}
\widehat{N}_{B O N D}^{A-B}=\frac{1}{4} \sum_{i, j}\left(1-\hat{\sigma}_{i} \hat{\sigma}_{j}\right) \tag{2}
\end{equation*}
$$

where the sum is taken over adjacent pairs of atoms. In turn, the coordination number of an atom in the nanoparticle is

$$
\begin{equation*}
\widehat{N}_{i}^{C O R}=\sum_{j} \hat{\sigma}_{j}^{2} \tag{3}
\end{equation*}
$$

where the sum is taken over sites adjacent to the site $i$. In the nanoparticle with $f c c$ structure corner atoms have coordination number 6, edge atoms -7 , atoms on $\{111\}$ terraces -9 and interior atoms 12. (Atoms on $\{100\}$ terraces with coordination number 8 are not considered here for simplicity.) Hence, one can express the quantities in (1) as operators in the following way:

$$
\begin{aligned}
& \widehat{N}_{C O R N E R}^{A}=-\sum_{i}\left(\hat{\sigma}_{i}+1\right)\left(\widehat{N}_{i}^{C O R}-7\right)\left(\widehat{N}_{i}^{C O R}-9\right)\left(\widehat{N}_{i}^{C O R}-12\right) / 36 \\
& \widehat{N}_{E D G E}^{A}=\sum_{i}\left(\hat{\sigma}_{i}+1\right)\left(\widehat{N}_{i}^{C O R}-6\right)\left(\widehat{N}_{i}^{C O R}-9\right)\left(\widehat{N}_{i}^{C O R}-12\right) / 20 \\
& \widehat{N}_{T E R R A C E}^{A}=-\sum_{i}\left(\hat{\sigma}_{i}+1\right)\left(\widehat{N}_{i}^{C O R}-6\right)\left(\widehat{N}_{i}^{C O R}-7\right)\left(\widehat{N}_{i}^{C O R}-12\right) / 36
\end{aligned}
$$

where sums are taken over all atoms. Consequently, the topological energy expression $E_{T O P}$ can be obtained by applying the following Hamiltonian to the system's wave function:

$$
\begin{equation*}
\widehat{H}_{T O P}=E_{0}+\varepsilon_{B O N D}^{A-B} \widehat{N}_{B O N D}^{A-B}+\varepsilon_{C O R N E R}^{A} \widehat{N}_{C O R N E R}^{A}+\varepsilon_{E D G E}^{A} \widehat{N}_{E D G E}^{A}+\varepsilon_{T E R R A C E}^{A} \widehat{N}_{T E R R A C E}^{A} \tag{7}
\end{equation*}
$$

Presently we do not see a practical advantage of the topological Hamiltonian in eq. (7) over the topological energy expression in eq. (1).

## 2. Dependence of calculated mixing energies on the basis set size

Smaller cut-off energies, $\epsilon_{\text {CUT }}$, were used in the electronic structure (DFT) calculations to construct topological energy expressions and to assess their accuracy, whereas the bigger cut-off energies of 415 eV could be recommended for VASP calculations involving C and O atoms (i.e. in studies of organic reactions catalyzed by transition metals). The differences between the mixing energies calculated with the two cut-off values are less than 1 meV per atom, which justifies the employment of the smaller cutoff in the present study.

Table S1. Comparison of the mixing energies per atom $E_{E S}^{M I X}$ of the most stable $\mathrm{Pd}_{70} \mathrm{X}_{70} \mathrm{NPs}(\mathrm{X}=\mathrm{Au}, \mathrm{Ag}$, $\mathrm{Cu}, \mathrm{Zn}$ ), obtained from density functional plane-wave calculations performed with the common ( 415 eV ) and reduced cut-off energy values $\epsilon_{\text {CUT }}$.

| X | $E_{E S}^{M I X}, \mathrm{meV}$ | $\epsilon_{\text {CUT }}, \mathrm{eV}$ |
| :--- | :--- | :--- |
| Au | -108.99 | 250.9 |
|  | -109.35 | 415 |
| Ag | -107.94 | 250.9 |
|  | -108.67 | 415 |
| Cu | -119.47 | 273.2 |
|  | -118.85 | 415 |
| Zn | -498.05 | 276.7 |
|  | -498.52 | 415 |

## 3. Chemical ordering in $\mathrm{Au}_{61} \mathrm{Pd}_{61}$ crystallite:

## Evaluation of method performance for less symmetric nanoparticles

In the present work we determine the chemical ordering in highly symmetric truncated octahedral NPs. In order to demonstrate that our method is not limited by any means to such "magic" particle shapes we also calculated the chemical ordering in $\mathrm{Au}_{61} \mathrm{Pd}_{61} \mathrm{NP}$ of only $C_{3 v}$ symmetry. The NP shape is derived from a Pd NP supported on $\mathrm{MgO}(100)$, the structure of which was optimized by some of us in Ref. 28 [ S. M. Kozlov, H. A. Aleksandrov, J. Goniakowski and K. M. Neyman, J. Chem. Phys., 2013, 139, 084701].


Figure S1. Surface (left) and interior (right) shells of the lowest-energy $\mathrm{Pd}_{61} \mathrm{Au}_{61}$ homotop according to density functional calculations. Pd atoms are displayed as cyan spheres, Au atoms - gold spheres.

Table S2. Comparison of descriptors $\varepsilon_{i}{ }^{a}$ in the topological expressions $E_{T O P}$ for $\operatorname{Pd}_{61} A_{u} u_{61}$ and $P^{2} d_{70} A u_{70} N P s$ with the precision and accuracy values (in meV ) and the number of structures used for the fitting, $N_{\text {FII }}$.

|  | $\mathrm{Pd}_{61} \mathrm{Au}_{61}$ | $\mathrm{Pd}_{70} \mathrm{Au}_{70}$ |
| :--- | :---: | :---: |
| $\varepsilon_{B O N D}^{P d-A u}$ | $-16_{-3}^{+2}$ | $-13_{-6}^{+4}$ |
| $\varepsilon_{C O R N E R}^{A u}$ | $-409_{-38}^{+27}$ | $-404_{-72}^{+76}$ |
| $\varepsilon_{E D G E}^{A u}$ | $-421_{-35}^{+51}$ | $-301_{-77}^{+52}$ |
| $\varepsilon_{\text {TERRACE }}^{A u}$ | $-320_{-45}^{+65}$ | $-200_{-64}^{+52}$ |
| $\varepsilon_{\text {LAYER }}$ | - | - |
| $N_{\text {FIT }}$ | 27 | 32 |
| precision, $\delta$ | 106 | 115 |
| accuracy, $\Delta E$ | 64 | 26 |

${ }^{a} 95 \%$ confidence intervals of $\varepsilon_{i}\binom{+\mathrm{k}}{-1}$ are also given.

As shown in Tables S2, S3 and Figure 1, the individual topological descriptors, the overall picture of interactions as well as the chemical ordering in the $\mathrm{Pd}_{61} \mathrm{Au}_{61} \mathrm{NP}$ are very similar to those obtained for the highly symmetric truncated octahedral $\mathrm{Pd}_{70} \mathrm{Au}_{70} N P$. The accuracy $\Delta E$ and precision $\delta$ values of the $E_{\text {TOP }}$ expressions for $\mathrm{Pd}_{61} \mathrm{Au}_{61}$ and $\mathrm{Pd}_{70} \mathrm{Au}_{70}$ are also very close. These findings strongly suggest that the method is applicable to reliably describe chemical ordering also in nanocrystallites with less symmetrical shapes. Notably, there are two different types of corner atoms with coordination numbers 5 and 6 in $\mathrm{Pd}_{61} \mathrm{Au}_{61}$. One may account for this difference by substituting $\varepsilon_{C O R N E R}^{A u} N_{C O R N E R}^{A u}$ by two separate terms $\varepsilon_{C O R N E R-5}^{A u} N_{C O R N E R-5}^{A u}$ and $\varepsilon_{C O R N E R-6}^{A u} N_{C O R N E R-6}^{A u}$. However, in this case the fitted values of $\varepsilon_{C O R N E R-5}^{A u}$ and $\varepsilon_{C O R N E R-6}^{A u}$ would be within 15 meV from each other, which is notably lower than the statistical inaccuracy of these parameters.

Table S3. Structural data of $\mathrm{Pd}_{61} \mathrm{Au}_{61}$ and $\mathrm{Pd}_{70} \mathrm{Au}_{70}$ homotops with the lowest energies $E_{E S} \cdot N_{B O N D}^{P d-A u}$ - the number of Pd-Au bonds; $N_{\text {CORNER }}^{A u}, N_{E D G E}^{A u}, N_{\text {TERRACE }}^{A u}, N_{S U R F A C E}^{A u}, N_{\text {INTERIOR }}^{A u}$ - the numbers of Au atoms in corner, edge, terrace, surface and interior positions, respectively. The first row for each NP shows the absolute numbers and the percentage of the possible maximum values is in the second row.

| NP | $N_{\text {BOND }}^{\text {Pd-Au }}{ }^{a}$ | $N_{\text {CORNER }}^{\text {Au }}{ }^{b}$ | $N_{\text {EDGE }}^{A u}$ | $N_{\text {TERRACE }}^{A u}$ | $N_{\text {SURFACE }}^{A u}{ }^{c}$ | $N_{\text {INTERIOR }}{ }^{\text {Au }}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Pd}_{61} \mathrm{Au}_{61}$ | $225 / 543$ | $18 / 18$ | $27 / 27$ | $16 / 43$ | $61 / 88$ | $0 / 34$ |
|  | $41 \%$ | $100 \%$ | $100 \%$ | $37 \%$ | $69 \%$ | $0 \%$ |
| $\mathrm{Pd}_{70} \mathrm{Au}_{70}$ | $260 / 636$ | $24 / 24$ | $24 / 24$ | $22 / 48$ | $70 / 96$ | $0 / 44$ |
|  | $41 \%$ | $100 \%$ | $100 \%$ | $46 \%$ | $73 \%$ | $0 \%$ |

[^0]
[^0]:    ${ }^{a}$ The percentage of $\mathrm{Pd}-\mathrm{Au}$ bonds is given with respect to the total number of bonds in the NP; ${ }^{b} N_{C O R N E R}^{A u}$ is the sum of Au atoms in 5-and 6-coordinated corner positions;
    ${ }^{c} N_{S U R F A C E}^{A u}=N_{C O R N E R}^{A u}+N_{E D G E}^{A u}+N_{T E R R A C E}^{A u}$ is the total number of Au atoms on the surface;
    ${ }^{d} N_{\text {INTERIOR }}^{A u}$ is the number of Au atoms not exposed on the NP surface.

