SUPPLEMENTARY MATERIAL

Additional considerations on the model and the data analysis

Scaling with d

We present here a simple argument for the scaling of the amplitude s with distance d, explaining the scaling seen for experiments and simulations in figure 3(d).

To estimate the d-dependency of the amplitude during a cycle, it is sufficient to describe the displacement of the central bead in one of the four subphases. We consider the simplified but physically equivalent situation of two interacting trapped beads at distance d. Initially the left bead, which represents the central bead in the experiment, is at rest and the right bead is out of its equilibrium position by a distance ϵ, as it would be if in the instant the trap has been shifted. The initial conditions are thus $x_L(0) = 0, x_R(0) = d$ and the equilibrium positions of the trapping potentials are $x_{0,L} = 0; x_{0,R} = (d - \epsilon)$. To a first approximation, ("zeroth order" in R/d) the left bead is still, and the position of the right bead follows a simple relaxation law

$$x_R^{(0)}(t) = \epsilon \left(e^{-\frac{t}{\tau_0}} - 1 \right) + d \quad . \tag{1}$$

This unperturbed solution can be used to estimate (by the force balance with the fluid) the source of force $F_R^{(0)} = -k_{\text{trap}}(x_R^{(0)} - d)$ applied by the right bead on the fluid during its relaxation. This can then be used into equation (1) of the main text for the left bead, giving

$$\tau_0 \, \dot{x}_L^{(1)} = x_L^{(1)} - \frac{3R}{2d} x_R^{(0)} \quad , \tag{2}$$

where $x_R^{(0)}$ appears as an external perturbation, and we approximated the distance between the beads with d, which is justified in the limit of large distances $d >> \epsilon$. With this assumption, the problem becomes linear and the solution to order R/d can be calculated as

$$x_L^{(1)}(t) = \frac{3R\epsilon}{2d}\frac{t}{\tau_0}e^{-\frac{t}{\tau_0}} \quad . \tag{3}$$

In turn, this solution could be used as a source for the equation for x_R, to obtain hierarchically the higher order contribution in R/d to its motion. The value of the peak of the central bead in the experiment s can be estimated by the maximum displacement of the left bead

$$x_{L,\mathsf{max}}^{(1)} = \frac{3R\epsilon}{2de} \quad . \tag{4}$$

This argument implies that the leading order scaling of each peak in a cycle, and hence of s, is $1/d$. The argument has the advantage of showing how the hydrodynamic interaction tensor comes into play explicitly after a trap switches its position.
In the same linear approximation, the coupled equations (eq.1) from the main text can even be solved directly in a straightforward way, and the resulting maximum displacement is:

$$x_{L,\mathsf{max}} = \frac{3R\epsilon}{2d}\frac{1}{\left(1+\frac{3R}{2d}\right)^{1+\frac{2d}{3R}}} \quad , \tag{5}$$

which has the same behavior in the limit of large d/R.

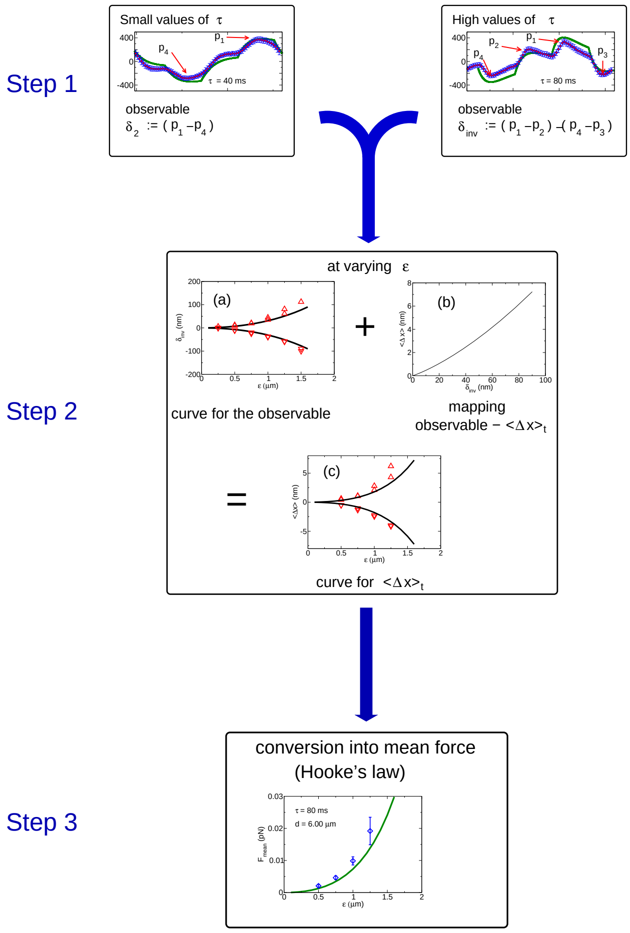
Strictly speaking, this result is applicable in the regime $\tau >> \tau_0$, in which the beads have the time to fully relax in the trap potentials. In the opposite limit, $\tau << \tau_0$, since the subcycle ends while the central bead is still moving away from the center of its trap, the maximum position can be estimated using the same solution, $x_L^{(1)}(t)$, by the position assumed by the central bead at the end of this subcycle, i.e. the instant $t = \tau$. Thus as

$$x_{L,\mathsf{max}}^{\prime(1)} = \frac{3R\epsilon}{2d}\frac{\tau}{\tau_0}e^{-\frac{\tau}{\tau_0}} \quad , \tag{6}$$

and the scaling with d is unaffected.

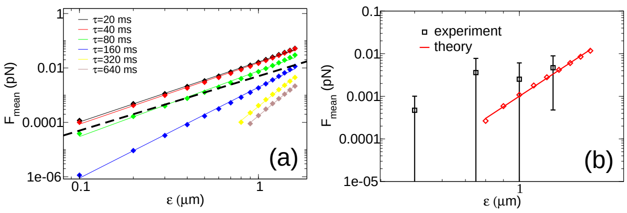
Relating the observables δ_2 and δinv with the mean force.

We summarise here how the procedure to relate the experimental data to the induced flow.



Supplementary Figure S1: **Converting observables into mean force.** This scheme illustrates the procedure used to obtain the mean force from the experiments, by means of simulations. Step 1: Because of the different shapes of the mean cycles for different values of τ, we define two different observables δ_2 and δinv to quantify the asymmetry in the displacements for the central bead. The peaks which enter in the definitions are indicated with red arrows. Step 2: (a) We analyze the mean cycle of experiments and simulations, extracting each observable at varying ϵ. We also calculate directly the temporal average $\langle \Delta x \rangle$ for the position of the central bead. (b) Comparing these results and eliminating the dependence from ϵ, we find that there exist a one to one mapping between each observable and the temporal average $\langle \Delta x \rangle$. (c) Using this relation the curve of each observable as a function of ϵ can be converted into a curve of the mean position as a function of ϵ. Step 3: using Hooke's law we convert the mean displacement into a mean force as a function of ϵ.

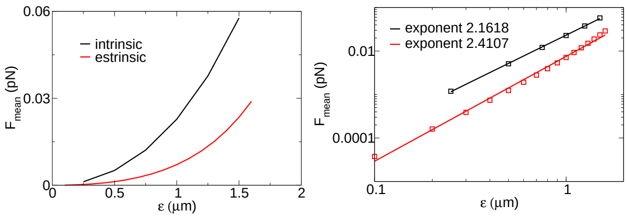
Scaling with τ



Supplementary Figure S2: **Scaling law for the mean force at varying τ.** (a) Plot on log-log scale of the simulated mean displacement from the equilibrium position of the central bead. The curve is a power law with different exponents for different values of τ. In our model the exponent varies, and increases monotonically for increasing τ. In the limit of small τ the mean displacement follows a power law with exponent close to 2 (dashed line), which resembles the behavior of the Golestanian swimmer. (b) Comparison between experimental and theoretical data for $\tau = 320$ ms on log-log scale. Due to the large error bars on the experimental curve, the determination of the exponent from the experiment is subject to large uncertainties.

Intrinsic Swimmer

Figure S3 shows a comparison of our model with simulations of an analogous intrinsic swimmer.



Supplementary Figure S3: **Comparison of the propulsive forces for extrinsic and intrinsic swimmers.** The extrinsic swimmer is studied experimentally and numerically in this work, whereas we can only study the intrinsic swimmer, actuated by two-state springs, numerically. The plot shows the mean force with varying ϵ, for simulations of the two models, with parameters $d = 6\mu$m $\tau = 80$ms. The difference between the two swimmers is only quantitative.