Supplementary Material (ESI) for Soft Matter

Supplementary Information

Lengthscale Dependence of Apparent Critical Exponents Determined by Vibration-corrected Two-particle Microrheology

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Vibrations and correlations combine additively in the correlation tensor

Equation 1 in the main text illustrates the calculation of the two particle displacement correlation tensor from particle displacements. Measured particle displacements suffer from the problem of being able to separate true particle motion, $\Delta r(t)$, from bulk motion due to vibrations and drift, $B(t)$, and the static tracking error, $\varepsilon(t)$. The equations below show that the true correlated motion and that due to bulk motion are separated additively in D_{rr}, and that the tracking error disappears in two-particle measurements. For clarity, the arguments t and τ are omitted from the equations.

$$D_{rr} = \left\langle \left(\Delta r^i + B + \varepsilon^i\right)\left(\Delta r^j + B + \varepsilon^j\right)\delta\left(r - R_{ij}\right)\right\rangle_{t,i\neq j} \qquad \text{E1}$$

$$D_{rr} = \left\langle \left(\Delta r^i \Delta r^j + B^2 + \varepsilon^i \varepsilon^j + B\left(\Delta r^i + \Delta r^j + \varepsilon^i + \varepsilon^j\right) + \varepsilon^i \Delta r^j + \varepsilon^j \Delta r^i\right)\delta\left(r - R_{ij}\right)\right\rangle_{t,i\neq j} \qquad \text{E2}$$

Assuming that the random displacements, the bulk motion and tracking errors are mutually independent:

$$\left\langle B\left(\Delta r^i + \Delta r^j + \varepsilon^i + \varepsilon^j\right)\right\rangle = 0,$$

$$\left\langle \varepsilon^i \Delta r^j + \varepsilon^j \Delta r^i \right\rangle = 0,$$

and further that the tracking error for different particles are not correlated:

$$\left\langle \varepsilon^i \varepsilon^j \right\rangle = 0.$$

Equation E2 simplifies to:

$$D_{rr} = \left\langle \left(\Delta r^i \Delta r^j + B^2\right)\delta\left(r - R_{ij}\right)\right\rangle_{t,i\neq j}$$

Bulk motion has no dependence on particle separation:

$$D_{rr} = \left\langle \Delta r^i(t,\tau)\Delta r^j(t,\tau)\delta\left(r - R_{ij}(t)\right)\right\rangle_{t,i\neq j} + \left\langle B^2(t,\tau)\right\rangle_t \qquad \text{E3}$$

The final term of Equation E3 is equivalent to the second term, $\beta(\tau)$, in equation 6 of the main text.

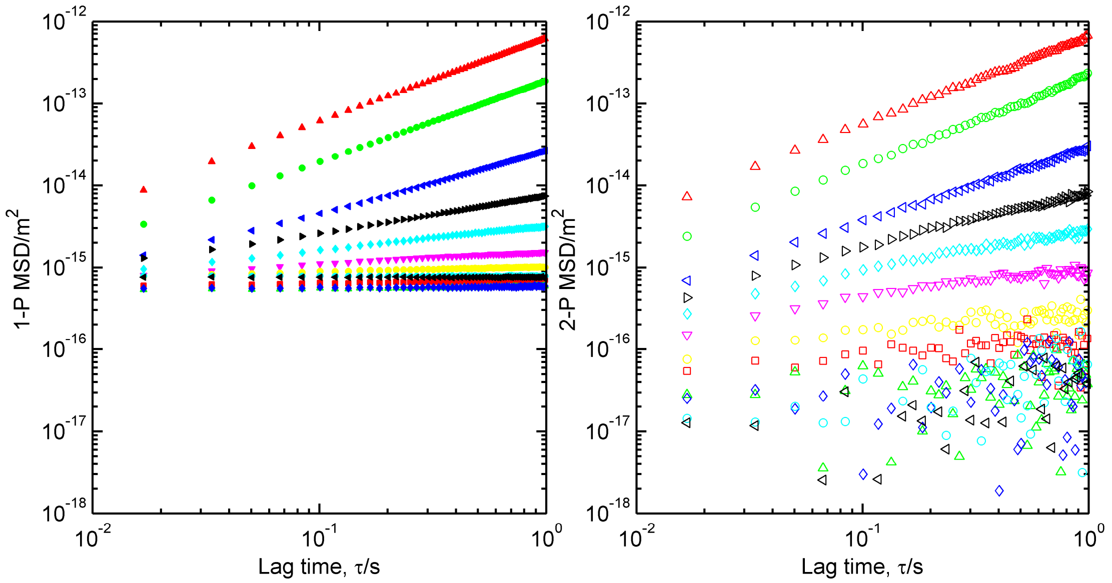

Fig. S1 Unshifted 1-P (left) and 2-P (right) MSDs for 5% BLG in 50:50 TFE:pH 7 phosphate buffer. Each curve represents a different quasi-static incubation time point. At early times (uppermost curves) there is little difference between 1-P and 2-P MSDs, however the lower limit of the long time 1-P MSDs is limited by the static tracking error, while the 2-P MSDs have no lower bound. The difference in curvature between 1-P and 2-P MSDs is more clearly seen in figure 4 of the main text.