Supplementary Information

Modeling the rupture of capillary liquid bridge between sphere and plane

Li Yang[ab], Yusong Tu [abc], and Haiping Fang*[ad]

[a] Shanghai Institute of Applied Physics, Chinese Academy of Sciences, P.O. Box 800-204, Shanghai, 201800, China. E-mail: fanghaiping@sinap.ac.cn

[b] Graduate School of the Chinese Academy of Sciences, Beijing, 100080

[c] Institute of Systems Biology, Shanghai University, Shanghai, 200444, China

[d] T-Life Research Center, Department of Physics, Fudan University, Shanghai, 200433, China

1. Model

The partial enlarged view of the liquid bridge separation process for Fig. 2 in the main text is shown in Fig. S1.

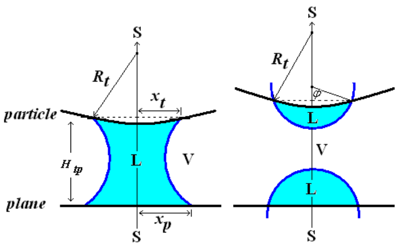

Figure S1. The partial enlarged view of the liquid bridge separation process for Fig. 2.

2. Total interfacial energy *W* of the liquid bridge

The particle-plane system has three phases (S, solid; L, liquid; V, vapor). The liquid bridge total interfacial energy *W* can be expressed as follows:

$$
\begin{aligned}
W &= W_{LV} + W_{SL} + W_{SV} \\
&= \gamma_{LV} A_{LV} + \gamma_{SL} A_{SL} + \gamma_{SV} A_{SV} \\
&= \gamma_{LV} A_{LV} + (\gamma_{SL-particle} A_{SL-particle} + \gamma_{SV-particle} A_{SV-particle}) \\
&\quad + (\gamma_{SL-plane} A_{SL-plane} + \gamma_{SV-plane} A_{SV-plane}) \\
&= \gamma_{LV} A_{LV} + [\gamma_{SL-particle} A_{SL-particle} + \gamma_{SV-particle} (C1 - A_{SL-particle})] \\
&\quad + [\gamma_{SL-plane} A_{SL-plane} + \gamma_{SV-plane} (C2 - A_{SL-plane})], \qquad \text{(S1)}
\end{aligned}
$$

where γ is the surface energy, *A* represents the interface area, *C1* and *C2* are arbitrary constants. Following the assumptions in references S1[S1] and S2[S2]

C1 = *C2* = 0,

W can be rewritten into

$$W = \gamma_{LV} A_{LV} + (\gamma_{SL-particle} - \gamma_{SV-particle}) A_{SL-particle} + (\gamma_{SL-pa\ln e} - \gamma_{SV-plane}) A_{SL-plane} . \quad (S2)$$

Taking the Young equation of $\gamma_{LV} \cos\theta = \gamma_{SV} - \gamma_{SL}$ into consideration, W can be evaluated as a function of the contact angle, surface tension and interface area

$$W = \gamma_{LV} A_{LV} - \gamma_{LV} \cos\theta_t A_{SL-particle} - \gamma_{LV} \cos\theta_p A_{SL-plane} . \quad (S3)$$

The surface areas are given by eqs. (S4), (S5), and (S6) as follows:

$$A_{SL-particle} = 2\pi R_t^2 (1 - \cos\beta) , \quad (S4)$$

$$A_{SL-pa\ln e} = \pi x_P^2 , \quad (S5)$$

$$A_{LV} = 2\pi \int_0^{y_t} x(y) \sqrt{1 + [f(y)^{/}]^2} dy . \quad (S6)$$

f(y) is the equation of the meniscus profile in the form of

$$f(y): \ (x - x_o)^2 + (y - y_o)^2 = r_{tp}^2 . \quad (S7)$$

Substituting eq. (S7) into eq. (S6), the surface area of liquid-vapor A_{LV} can be rewritten as

$$A_{LV} = 2\pi r_{tp} \{ x_o [\pi - (\beta + \theta_t + \theta_p)] - y_t \} . \quad (S8)$$

The total W is then given by substituting eqs. (S4), (S5) and (S8) into eq. (S3),

$$W = \gamma_{LV} 2\pi r_{tp} \{ x_o [\pi - (\beta + \theta_t + \theta_p)] - y_t \}$$

$$- 2\pi \gamma_{LV} \cos\theta_t R_t^2 (1 - \cos\beta) - \pi \gamma_{LV} \cos\theta_p x_P^2 . \quad (S9)$$

Thus, we arrive at eq. (7) in the main text.

3. The interfacial energy *U* of the two droplets

The interfacial energy of the two droplets is the sum of the energy of the liquid-vapor (LV), liquid-solid (LS) and solid-vapor (SV) interfaces,

$$U_{droplet} = U_{droplet1} + U_{droplet2}$$

$$= \gamma_{LV-droplet1} A_{LV-droplet1} + \gamma_{SL-droplet1} A_{SL-droplet1} + \gamma_{SV-droplet1} A_{SV-droplet1}$$

$$+ \gamma_{LV-droplet2} A_{LV-droplet2} + \gamma_{SL-droplet2} A_{SL-droplet2} + \gamma_{SV-droplet2} A_{SV-droplet2} . \quad (S10)$$

(a) Case $\theta_t \neq 0^o$ and $\theta_p \neq 0^o$

Considering the rupture of a liquid bridge, we assume that the shape of the two droplets is spherical ($\varphi = \theta_t = \theta_p$). The volume of a single droplet is

$$V_{droplet} = \pi R^3 (2 - 3\cos\theta_t + \cos^3\theta_t)/3 \ . \tag{S11}$$

The interfacial energy of a single droplet is

$$U_{droplet\,1} = \gamma_{LV} A_{LV} + \gamma_{SL} A_{SL} + \gamma_{SV} A_{SV}$$

$$= \gamma_{LV} \pi R^2 (2 - 3\cos\theta_t + \cos^3\theta_t) \ . \tag{S12}$$

Substituting eq. (S11) into eq. (S12), the expression of $U_{droplet\ 1}$ becomes

$$U_{droplet\,1} = \gamma_{LV} [9\pi(2 - 3\cos\theta_t + \cos^3\theta_t)]^{1/3} V_{droplet\,1}{}^{2/3} \ . \tag{S13}$$

For convenience, we introduce *P1* and *P2* by defining

$$P_1 = [9\pi(2 - 3\cos\theta_t + \cos^3\theta_t)]^{1/3}, \tag{S14}$$

$$P_2 = [9\pi(2 - 3\cos\theta_p + \cos^3\theta_p)]^{1/3}. \tag{S15}$$

Consequently, $U_{droplet\ 1}$ can be expressed as a function of P_1,

$$U_{droplet\,1} = \gamma_{LV} P_1 V_{droplet\,1}{}^{2/3} \ . \tag{S16}$$

With the identical methods, we can get

$$U = \gamma_{LV} P_1 V_{droplet\,1}{}^{2/3} + \gamma_{LV} P_2 V_{droplet\,2}{}^{2/3} \ . \tag{S17}$$

Using the identities $K = (\frac{P_1}{P_2})^3$ and $V = V_{droplet\,1} + V_{droplet\,2}$, U can be finally rewritten as follows:

$$U = \gamma_{LV} P_2 V^{2/3} (1 + K)^{1/3} . \tag{S18}$$

Thus, we arrive at eq. (8) in the main text.

(b) Case $\theta_t = \theta_p = 0^{\circ}$

Using the calculation methods as case (a)**,** we assume that shape of the two droplets is circular cone in the perfect wetting condition ($\varphi = \beta$), and that the height of the droplets are $H_{tp}/2$, the final expression for U is now given by

$$U = \gamma_{LV} A_{LV} + \gamma_{SL} A_{SL} + \gamma_{SV} A_{SV}$$

$$= \gamma_{LV} \pi [x_t \sqrt{x_t{}^2 + H_{tp}{}^2} - x_t{}^2 + x_P \sqrt{x_P{}^2 + H_{tp}{}^2} - x_P{}^2] \ . \tag{S19}$$

Therefore, we get eq. (9) in the main text.

References

S1. X. Pepin, D. Rossetti, S. M. Iveson and S. J. R. Simons, *Journal of Colloid and Interface Science*, 2000, **232**, 289-297.

S2. P. Lambert, *Capillary forces in microassembly: modeling, simulation, experiments, and case study*, Springer Verlag, 2007.