Supplementary Material (ESI) for Soft Matter

Supplementary information to:

Microtubule nanospool formation by active self-assembly is not initiated by thermal activation

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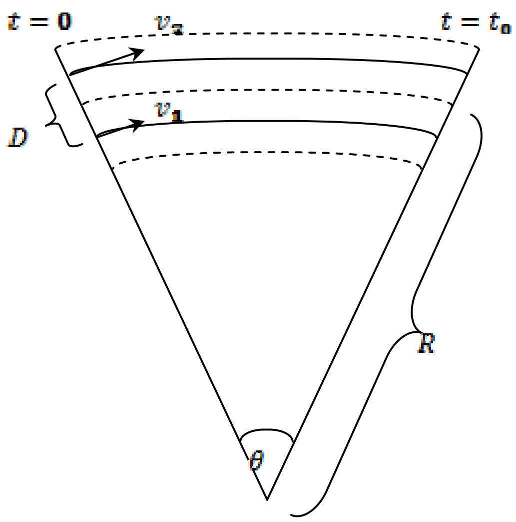
Spooling formation due to bundle velocity difference (Ofer Idan)

Assume two microtubules of different velocities meet. If the direction of movement is the same, and the speed is different, the bundle will start bending in the direction of the slower microtubule:



This implies:

$$v_2 t_0 = \theta\left(R + \frac{D}{2}\right) \tag{1}$$

$$v_1 t_0 = \theta\left(R - \frac{D}{2}\right) \tag{2}$$

$$\Rightarrow \Delta v = \frac{\theta D}{t_0} = \omega D \tag{3}$$

We can assume the velocity along the bundle is continuous, so that

$$v(R) = \frac{v_1 + v_2}{2} \tag{4}$$

The relation between radius and velocities can be inferred:

$$\frac{v(R)}{R} = \omega$$

$$\Rightarrow R = \frac{v(R)}{\omega} = \frac{D \cdot v(R)}{\Delta v} = \frac{D \cdot (v_1 + v_2)}{2\Delta v} \qquad (5)$$

We would like to calculate the probability of two microtubules would meet with a speed difference of Δv . Assume that the speed distribution of a single microtubule is normal with mean μ and variance σ^2.

The distribution of the difference will also be normally distributed:

$$dP_{X-Y}(\Delta v) = \frac{1}{\sqrt{4\pi\sigma^2}} e^{-\left[\frac{\Delta v^2}{4\sigma^2}\right]} d\Delta v \qquad (6)$$

Variable change for a probability density function:

$$f_X(x) = \frac{1}{\sqrt{4\pi\sigma^2}} e^{-\left[\frac{x^2}{4\sigma^2}\right]}$$

$$R = y = h(x) = \frac{Dv}{x}, h^{-1}(y) = \frac{Dv}{y}$$

$$h'(x) = -\frac{Dv}{x^2} \neq 0, h \text{ is monotonic}$$

$$\Rightarrow f_Y(y) = \begin{cases} -\frac{1}{h'(h^{-1}(y))} \cdot f_X(h^{-1}(y)), y \in h(\mathbb{R}) \\ 0 \text{ else} \end{cases}$$

$$|(h^{-1})'(y)| \cdot f_X(h^{-1}(y)) = \left|-\frac{Dv}{y^2}\right| \cdot \frac{1}{\sqrt{4\pi\sigma^2}} e^{-\left[\frac{Dv}{2y\sigma}\right]^2}$$

$$\Rightarrow f_Y(R) = \frac{Dv}{R^2} \cdot \frac{1}{\sqrt{4\pi\sigma^2}} e^{-\left[\frac{D^2v^2}{4R^2\sigma^2}\right]}$$

Now do the same for the radius and the circumference:

$$f_X(x) = \frac{Dv}{x^2} \frac{1}{\sqrt{4\pi\sigma^2}} e^{-\left[\frac{Dv}{2x\sigma}\right]^2}$$

$$C = y = h(x) = 2\pi x, h^{-1}(y) = \frac{y}{2\pi}$$

$$h'(x) = 2\pi \neq 0, h\ is\ monotonic$$

$$\Rightarrow f_Y(y) = \frac{1}{2\pi}\frac{Dv}{\left(\frac{y}{2\pi}\right)^2}\frac{1}{\sqrt{4\pi\sigma^2}}e^{-\left[\frac{Dv}{2\frac{y}{2\pi}\sigma}\right]^2} = \frac{2\pi Dv}{y^2}\frac{1}{\sqrt{4\pi\sigma^2}}e^{-\left[\frac{\pi Dv}{y\sigma}\right]^2}$$

$$\Rightarrow P(C) = \frac{2\pi Dv}{C^2}\frac{1}{\sqrt{4\pi\sigma^2}}e^{-\left[\frac{\pi Dv}{C\sigma}\right]^2}$$

The integral of this function from 0 to infinity is ½, but since we get the same circumference for 2 different values of Δv , this must be multiplied by 2, giving 1.

Therefore:

$$P(C) = \frac{4\pi Dv}{C^2}\frac{1}{\sqrt{4\pi\sigma^2}}e^{-\left[\frac{\pi Dv}{C\sigma}\right]^2}$$