Electronic Supplementary Material (ESI) for Soft Matter

"Plastron Induced Drag Reduction and Increased Slip on a Superhydrophobic Sphere", G. McHale, M.R. Flynn and M.I. Newton.

Appendix A – Streamfunctions

Ferreira *et al.* give the streamfunction solutions for single and multiple compound droplets in a creeping flow regime.[1] For a fluid-encapsulated fluid core of radius *b*, in axisymmetric creeping flow the three stream functions are,

Core ($0 \le r \le b$)

$$\psi_3 = \frac{U_\infty r^2 \sin^2\theta}{2}\left(\frac{\eta_{12}\tilde{S}}{\tilde{\Delta}}\right)\left(1-\frac{r^2}{b^2}\right) \quad \text{(A1)}$$

where $\varepsilon = b/a$ and a is the radius of the fluid core together with the encapsulating fluid (see fig. 1 of the manuscript). Here U_∞ is the free stream velocity at large r and $\eta_{12} = \eta_1/\eta_2$ is the ratio of the viscosity of the sheathing fluid (phase 1) to the external fluid (phase 2).

Sheathing fluid ($b \le r \le b/\varepsilon$)

$$\psi_1 = \frac{U_\infty r^2 \sin^2\theta}{2}\left(\frac{1}{1-\varepsilon^5}\right)\left(\frac{G(\varepsilon)}{\tilde{\Delta}}\right)\left[2\eta_{12}\left(\frac{\varepsilon^2 r^2}{b^2} - T + \frac{Tb}{r} - \frac{\varepsilon^2 b^3}{r^3}\right) + \eta_{32}(1-\varepsilon)\left(\frac{\tilde{U}\varepsilon^2 r^2}{b^2} - \tilde{V} + \frac{\tilde{W}b}{\varepsilon r} - \frac{\tilde{X}b^3}{r^3}\right)\right] \quad \text{(A2)}$$

where $\eta_{32} = \eta_3/\eta_2$ is the ratio of the viscosity of the core fluid (phase 3) to the external fluid (phase 2).

External fluid ($b/\varepsilon \le r$)

$$\psi_2 = \frac{U_\infty r\left(r - \frac{b}{\varepsilon}\right)\sin^2\theta}{2}\left(\frac{1}{\tilde{\Delta}}\right)\left[2\eta_{12}F(\varepsilon) + \eta_{32} + \eta_{12}\left(2\eta_{12}G(\varepsilon) + \eta_{32}F(\varepsilon)\right)\left(1-\frac{b}{\varepsilon r}\right)\left(2+\frac{b}{\varepsilon r}\right)\right] \quad \text{(A3)}$$

where

$$\tilde{\Delta} = \eta_{32} + 4\eta_{12}^2 G(\varepsilon) + 2\eta_{12}(1+\eta_{32})F(\varepsilon) \qquad \text{(A4)}$$

and

$$\tilde{S} = \frac{(\varepsilon^2 + 3\varepsilon + 1)}{(1-\varepsilon)(4\varepsilon^2 + 7\varepsilon + 4)} \qquad \text{(A5a)}$$

$$T = \frac{(1-\varepsilon^5)}{(1-\varepsilon)} = \varepsilon^4 + \varepsilon^3 + \varepsilon^2 + \varepsilon + 1 \qquad \text{(A5b)}$$

$$\tilde{U} = (\varepsilon + 2) \qquad \text{(A5c)}$$

$$\tilde{V} = (3\varepsilon^3 + 6\varepsilon^2 + 4\varepsilon + 2) \qquad \text{(A5d)}$$

$$\tilde{W} = \varepsilon(2\varepsilon^3 + 4\varepsilon^2 + 6\varepsilon + 3) \qquad \text{(A5e)}$$

$$\tilde{X} = (2\varepsilon + 1) \qquad \text{(A5f)}$$

In the limit of a solid core (i.e. $\eta_{32}\to\infty$), these equations reduce to,

$$\psi_3 = 0 \qquad 0 \le r \le b$$

(A6)

$$\psi_1 = \frac{U_\infty r^2 \sin^2\theta}{2}\frac{G(\varepsilon)}{1+2\eta_{12}F(\varepsilon)}\left(\frac{1}{T}\right)\left[\frac{\tilde{U}\varepsilon^2 r^2}{b^2} - \tilde{V} + \frac{\tilde{W}b}{\varepsilon r} - \frac{\tilde{X}b^3}{r^3}\right] \qquad b \le r \le b/\varepsilon$$

(A7)

$$\psi_2 = \frac{U_\infty r\left(r - \frac{b}{\varepsilon}\right)\sin^2\theta}{2}\left(\frac{1}{1+2\eta_{12}F(\varepsilon)}\right)\left[1 + \eta_{12}\left(1 - \frac{b}{\varepsilon r}\right)\left(2 + \frac{b}{\varepsilon r}\right)F(\varepsilon)\right] \qquad b/\varepsilon \le r$$

(A8)

Note that ψ_1 vanishes at both b and b/ε, and ψ_2 vanishes at b/ε (see fig. 3 of the manuscript). In the limit $r\to\infty$, the stream function $\psi_2\to 0.5U_\infty r^2\sin^2\theta$, which is the expected free stream value.

Reference

1. J. M. Ferreira, A. A. Soares and R. P. Chhabra, *Fluid Dyn. Res.*, **2003** *32*, 210-215.