Electronic Supplementary Material (ESI) for Soft Matter

Appendix 1

Materials characterisation

Porosity

Prior to the wicking experiments, a piece of dry sample was cut, pre-weighted, and the geometrical parameters were measured. The sample thickness h was measured by sandwiching the sample between glass slides; the sample width and length was measured with a ruler. Using the measured parameters, the cross-sectional area $A_{||}$ of the sample was calculated. The sample was soaked in water until completely saturated, removed from the water reservoir, left vertically hanging for few a moments to let the droplets drip and weighed again. The sample porosity was obtained by using the following formula:

$\varepsilon_{\perp}=V_p/(hA_{||}) = (m^+ - m^-)/(\rho_l h A_{||})$ where V_p is the pore volume, m^- and m^+ are masses of dry and wet samples respectively, and ρ_l is water density. Using this procedure, we measured the linear densities of dry and wet samples and obtained the following values $\rho^-=5\cdot10^{-4}$ kg/m and $\rho^+= 5.27\cdot10^{-3}$ kg/m. Then the total weight of the dry 17 cm long sample was estimated as $m^- = 0.085$ g and the wet one as $m^+ = 0.895$ g. Finally, using these parameters, the porosity of the sample was estimated as $\varepsilon_{\perp}= 0.91$. Comparing the experimental data with the theoretical predictions, we assumed that porosities are identical for wet and dry parts of the sample. We neglected the fibrous network change upon liquid invasion and set the ratio of pore volume to the material volume to be the same for dry and wet parts of the material.

Effective pore size

As it is be shown below, the driving pressure in the experiments consists of two parts. One is a hydrostatic pressure, dictated by the shape of the sample. Another is a capillary pressure, which depends on the effective pore radius in the material, R_p. To determine this radius, the upward wicking experiment was conducted and the liquid front position was plotted as a function of time. In the experiments, the shape of the wetting front is not strictly uniform. Instead of averaging the fluctuations of the front profile along the sample width it was decided to introduce the effective linear front based on the weight of liquid entered the sample.

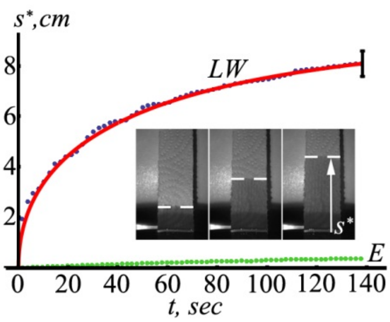

Fig.A1 Front position in the wicking experiment as a function of time: data are fitted with Lucas-Washburn law (solid line, LW). The circles (line E) correspond to the evaporation rate in centimeters of water column, i.e. it is a ratio of the evaporated water volume to the area of the sample cross-section. The Error bar indicates the maximum standard deviation within the three experiments. The insert defines the position of the wetting front

A Petri-dish filled with water was positioned on the scales (Sartorius 210S) connected to the computer. A strip of a paper towel was fixed vertically above the water, and lowered until it touched the surface. From that moment, the reading of the scales was recorded and the difference between the initial and current weights of the Petri dish was interpreted as the weight of the liquid absorbed by the sample at the given moment of time. Each experiment was conducted during 10 minutes and analyzed using the mass measuring software (Data Master 2003). The weight change due to water evaporation was found to be relatively small at the initial stages of liquid absorption (up to 2 minutes, see Fig. 2). The position of wetting front s^* at time t was calculates as $s^* = m/(\rho_l h A_{||}\varepsilon_{\perp})$, where m is the mass of the liquid that was absorbed by the paper towel at time t. During the experiments, the humidity was 22±3% and the temperature was 21±2 °C. The insert on Fig.A1 shows front position $s^*(t)$ in a vertically hanging sample. The experimental curve was analysed using the Lucas-Washburn law [14, 15]

$$t = -(A/B)\cdot\ln(1 - s^*/A) - s^*/B. \qquad \text{(A.1)}$$

where $A = P_c/\rho_l g$ and $B = \rho_l g k/\varepsilon\eta$. The coefficients A and B are considered as adjustable parameters that provide the best fit for the experimental data with eq. (A.1). Once determined, the capillary pressure P_c and the permeability k were calculated as $P_c = A\rho_l g$ and $k = B\varepsilon\eta/\rho_l g$ [8]. The effective pore radius R_p was then found from the defining equation for the capillary pressure as $R_p=2\gamma/P_c$, where γ represents the water surface tension. The following values were obtained from three experiments: A=9.66±0.38, B=0.087±0.007, P_c=946±30 Pa, R_p=152±6μm, k=2.3±0.18 $10^{-10}\cdot m^2$. Based upon the estimated pore radius, the calculated Jurin length for water was found to be approximately 9.7 cm.

These measurements of effective pore radius were confirmed by using the bubble point test (Capillary Flow Porometer CFP-1100-AXES, PMI)on the same paper towels. The bubble point test gave R_p = 170 ± 35 μm average radius, which is in the same range as that obtained in the wicking experiments.

Appendix 2

As it was said, it is convenient to express the flow potential along the liquid column as a following sum $\Phi(l) = P_l + \rho_l g y^+(s + s_0, t)$. On the other hand the flow potential is a linear function of the length l: $\Phi = al + b$. We introduced the flow potential on the horizontal and sagged part separately.

Potential on the horizontal part of the sample was introduced as $\Phi_1 = a_1 l + b_1$. From the boundary conditions it follows that the pressure at the liquid source with $l = 0$ and vertical coordinate $y^+ = H$ and is equal to zero $P_l = 0$, then $b_1 = \rho_l g H$. Potential on the sagged part of the sample $\Phi_2 = a_2 l + b_2$. On the wicking front with , $y = y^+(s^*)$ and $l = s^* + s_0$ the pressure is equal to the capillary pressure, $P_l = -P_c$, then $a_2(s^* + s_0) + b_2 = -P_c + \rho_l g y^+(s^*)$.

At the suspension point $l = s_0$ the values of potentials $\Phi_1 = \Phi_2$ and flow rates $q_1 = q_2$ should be the same. This gives the following relations between the potential constants: $a_2 s_0 + b_2 = a_1 s_0 + b_1$ and $a_1 = a_2 = a$, thus $b_1 = b_2 = b$. Therefore, from the boundary conditions it follows that $a = -\left(P_c + \rho_l g\left(H - y^+(s^*)\right)\right)/(s^* + s_0)$. Thus, the Darcy's law on the wicking front $l = s^* + s_0$ is:

$$\varepsilon_\perp \cdot \frac{ds^*}{dt} = -(k/\eta)\cdot\left(-P_c + \rho_l g\left(y^+(s^*, t) - H\right)\right)/(s^* + s_0), \tag{A.2}$$

and corresponding equation for the flow potential:

$$\Phi = -\left(P_c + \rho_l g\left(H - y^+(s^*)\right)\right)\frac{s}{(s^* + s_0)} + \rho_l g H \tag{A.3}$$

and for pressure:

$$\Phi = -\left(P_c + \rho_l g\left(H - y^+(s^*)\right)\right)\frac{s}{(s^* + s_0)} + \rho_l g\left(H - y(s)\right) \tag{A.4}$$

Appendix 3

Problem formulation

The shape of a freely hanging fabric is controlled by the weight distribution over the fabric. In order to observe the effect of this distribution, we write the force balance and momentum balance equations[25] as

$$d\vec{F}/ds + \rho\vec{g} = 0\,,\ (d\vec{R}/ds)\times\vec{F} + \vec{R}(s)\times\left(d\vec{F}/ds + \rho\vec{g}\right) = 0\,,$$

where $\vec{F}$ is an effective tensile force acting at each point of the sagged fabric, $\rho = \rho(s)$ is the linear mass density of the fabric, s is the arc length, $\vec{g}$ is acceleration due to gravity, and $\vec{R}$ is a radius-vector emanating from an arbitrarily chosen center of coordinates and directed to point s (see Fig.A2). The arc length is measured from the point with Cartesian coordinates (D,H) where the liquid is applied to the fabric. From the momentum balance equation it immediately follows that the effective tension is directed along the tangent-vector $\vec{\tau} = d\vec{R}/ds$.

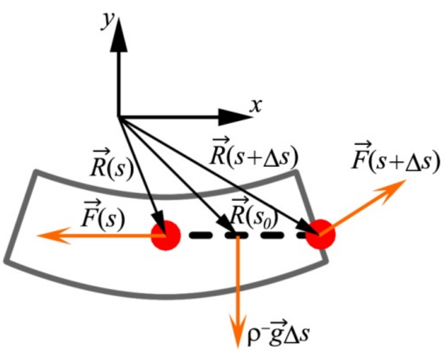

Fig.A2 The force balance for an elementary piece of the sample of arclength ds, ρ^- g – is the weight per unit length of the sample, $\vec{F}(s)$, $\vec{F}(s + \Delta s)$, are the stresses acting at the edges of the element Δs. $\vec{R}(s)$ and $\vec{R}(s + \Delta s)$, $\vec{R}_0(s)$ are the radii-vectors measured from the center of coordinates to the points of force application.

Therefore, the Cartesian components of the tensile force is written as $F_x = F dx/ds$, $F_y = F dy/ds$, where $F = F(s)$ is yet unknown magnitude of the tensile force. From this representation, the force balance equations are

$$\frac{d}{ds}F\frac{dx}{ds} = 0, \qquad \frac{d}{ds}F\frac{dy}{ds} = \rho(s)g\,, \tag{A.5}$$

where arc length s is defined through the following differential equation

$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \tag{A.6}$$

The density of a fabric possessing the wet and dry parts is a step-function,

$$\rho(s) = \rho^+,\ s < s^* \quad \text{and} \quad \rho(s) = \rho^-,\ s > s^*, \tag{A.7}$$

where ρ^+ and ρ^- are the densities of dry and wet parts, respectively, and the position of wetting front $s = s^*(t)$ is unknown in advance and must be found as a part of the solution.

Appendix 4

The eq. 13 can be re-written in the differential form:

$$\frac{(\bar{s}^*_{n+1} - \bar{s}^*_n)}{\bar{t}^*_{n+1} - \bar{t}^*_n} = \left(P_c/(\rho_l g L) + \bar{H} - \bar{y}^+(\bar{s}^*_n, \bar{t}^*_n)\right)/(\bar{s}^*_n + \bar{s}_0) \tag{A.8}$$

In order to eliminate the singularity as $\bar{s}^*_n -> 0$ in eq. A.8, we used the Lucas-Washburn approximation $\bar{s}^*(\bar{t}_0) \approx \sqrt{2\bar{t}_0 \cdot P_c/(\rho_l g L) + \bar{s}_0^{-2}} - \bar{s}_0$ with $\bar{t}_0 \approx 10^{-5}$. Choosing the time step $\bar{t}^*_{n+1} - \bar{t}^*_n$ so that the obtained increment of the front position $\bar{s}^*_{n+1} - \bar{s}^*_n$ would not exceed 0.001, we solved eq. A.8 with respect to $\bar{s}^*_{n+1}$. When the current front position is known, one can determine $\bar{y}^+(\bar{s}^*_{n+1}, \bar{t}^*_{n+1})$ numerically using the model presented in the Ref.[18] Then the procedure is repeated.