

Motion Induced by Asymmetric Enzymatic Degradation of Hydrogels

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Supplementary Material

S1. Degradation-induced motion of beads

Supplementary Movie 1: Motion of beads, gelatin, and trypsin in a gradient of trypsin concentration. The data was acquired at 12 frames per minute. The movie plays at 360x speed up. At time = 0, the right edge of the gelatin was exposed to a solution of 0.5 mg/mL trypsin in digested gelatin. Trypsin diffused into and degraded the gelatin, and beads moved up the trypsin gradient. (Top panel) Trans-illuminated image of beads. (Middle panel) Green fluorescence of gelatin labeled with AF488. (Bottom panel) Red fluorescence of trypsin labeled with AF647. Beads are 6 μm diameter, embedded in 5% gelatin.

S2. Transport of enzyme through gel

If the transport of the enzyme is purely diffusive and our sample is translationally invariant in y , then the concentration profile of enzyme, $c(x, t)$ is governed by the 1-D diffusion equation:

$$\frac{\partial c(x, t)}{\partial t} = D \frac{\partial^2 c(x, t)}{\partial x^2} \quad (\text{S1})$$

where D is the diffusion coefficient of enzyme through the gel. If the gel resides in a semi-infinite plane $x \geq 0$, the solution to Eq. S1 with the initial condition $c(x, 0) = c_0 \theta(-x)$ yields the following functional form for the dimensionless mean position of the enzyme, $\langle \tilde{x}_{\text{enz}}(t) \rangle$, over a region $x \in [0, x_f]$:

$$\langle \tilde{x}_{\text{enz}}(t) \rangle = \frac{Dt * \text{erf}\left(\frac{x_f}{2\sqrt{Dt}}\right) + \frac{1}{2}x_f \left(-2\sqrt{\frac{Dt}{\pi}} \exp\left(-\frac{x_f^2}{4Dt}\right) + x_f \text{erfc}\left(\frac{x_f}{2\sqrt{Dt}}\right)\right)}{2x_f \sqrt{\frac{Dt}{\pi}} \left(1 - \exp\left(-\frac{x_f^2}{4Dt}\right)\right) + x_f \text{erfc}\left(\frac{x_f}{2\sqrt{Dt}}\right)} \quad (\text{S2})$$

where $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ and $\text{erfc}(x) = 1 - \text{erf}(x)$.