Electronic Supplementary Material (ESI) for Soft Matter

Supplementary Material

A. Evaluation of substrate Young modulus from AFM measurements

Using the experimental correlations between force and deformation from AFM measurements, Young modulus for the substrate can be estimated using Hertz contact theory, for the specific case of contact between a rigid sphere, i.e. AFM probes (with radius *R*), and a deformable material, whose contact is a circular area (radius *a* – see Figure 2 in the main text). According to Hertz contact theory (see assumptions in the Section II), the applied force, *F*, is related to substrate deformation, *δ*, by a non-linear expression [17]:

$$F = \frac{4}{3} E^* R^{1/2} \delta^{3/2} \qquad \text{(S1)}$$

where E^* is a function of the deformable material Young modulus, E_S, and Poisson ratio, ν_S (nominal value of 0.5 was used for PDMS):

$$\frac{1}{E^*} = \frac{1-\nu_S^2}{E_S} \qquad \text{(S2)}$$

Note that the exponent for the substrate deformation, *δ,* predicted by the Hertz contact theory is 3/2 (see Eq. 3) and is equal to the exponent in the experimentally derived correlations for both hard and soft PDMS (see Eqs. 1 and 2, respectively). A comparison between Eq. 3 and experimental correlations (Eq. 1 and 2) allows to estimate values of Young modulus, *E*, for hard and soft substrates.

B. Details of Hertz contact theory and elastic foundation model

The general Hertz contact theory (see [17] for details) is based on the contact between a spherical body and a flat substrate, both deformable. Hertz contact theory assumes the following: no adhesion between the interacting particles; the interacting surfaces are frictionless; and the deformation is small, i.e. radius of contact area, a, and substrate deformation, δ, are much smaller than sphere radius, R. It can be shown that the correlation between force and deformation is the one reported in Eq. 3 of the main text, where E^* is calculated using sphere (subscript SP) and substrate (S) elastic properties as:

$$\frac{1}{E^*} = \frac{1-\nu_S^2}{E_S} + \frac{1-\nu_{SP}^2}{E_{SP}} \tag{S3}$$

Note that Eq. S3 reduces to:

$$\frac{1}{E^*} = \frac{1-\nu_S^2}{E_S} \tag{S4}$$

in case of perfectly rigid sphere ($E_{sp} \rightarrow \infty$).

The elastic foundation model can be used to analyze the contact between a rigid spherical body and an elastic substrate, ideally composed of an infinite number of springs with length, h, and elastic modulus, k. The applied force between the rigid sphere and the substrate is:

$$F_{EFM} = \pi \frac{k}{h} R\delta^2 \tag{S5}$$

For both models, the drop impact force, F, can be estimated assuming δ equal to the measured substrate deformation and applying either Eq. (Hertz contact theory) or Eq. S5 (elastic foundation model).

The analytical models have been calibrated through the contact of a steel sphere onto the soft PDMS substrate, introducing a corrective factor to fit experimental data. Since the steel sphere Young modulus is several orders of magnitude higher than that of hard and soft PDMS substrates analyzed in the present study, the second term on the right side of Eq. S3 becomes negligible compared to the first one, and Eq. S3 reduces to Eq. S4, where E^* is only function of substrate mechanical properties.

The calibration tests with steel spheres (mass of 1 and 5.6g) on the substrate led to definition of correlations given in Eqs. 5 and 6 in the paper, which include a correction factor.

C. Details of the numerical model

As initial condition for numerical simulations (at $t=0$), zero deformation for the entire geometry was applied. Considering Figure 6 in the paper, the following boundary conditions were applied: load due to drop impact on AB, free displacement on BC, no vertical displacement on BC, and axial symmetry on line AD. The drop impact load was applied assuming at each time step a uniform pressure over the drop-solid interface, defined by its radius, a, and zero pressure elsewhere; combining Eqs. 9 and 10, the applied pressure results in:

$$p=\begin{cases}\bar{p}\rho_d V^2 = 1.7\exp\left(-3.1tV/D\right)\rho_d V^2 & \text{for } r<a \\ 0 & \text{for } r\geq a\end{cases} \tag{S6}$$

Note that pressure at the impact point was calculated from Eq. 9 (fitting of numerical simulations) and not Eq. 8 (potential flow theory). This choice was made to avoid numerical instabilities, because Eq. 8 has a singularity for $t=0$.

The non-dimensional simulation time is equal to one ($\bar{t}=1$), which correspond to convective time, t_{conv} (range from 0.77 to 1.68ms). This simulation time is sufficient to capture the maximum deformation at the drop impact point. Although the assumptions for the radius of the wetted spot

(Eq. 10) and on pressure (Eq. 9) are only strictly valid for $\bar{t} < 1/2$, these expressions were used for simplicity also for $\bar{t} > 1/2$, because maximum impact force is reached at an earlier stage ($\bar{t} = 0.23$, see Figure S1); the eventual poor accuracy on the force prediction for $\bar{t} > 1/2$ would be only related to the force decay to zero (not of interest in this study). The execution time is approximately 50s on a desktop computer.

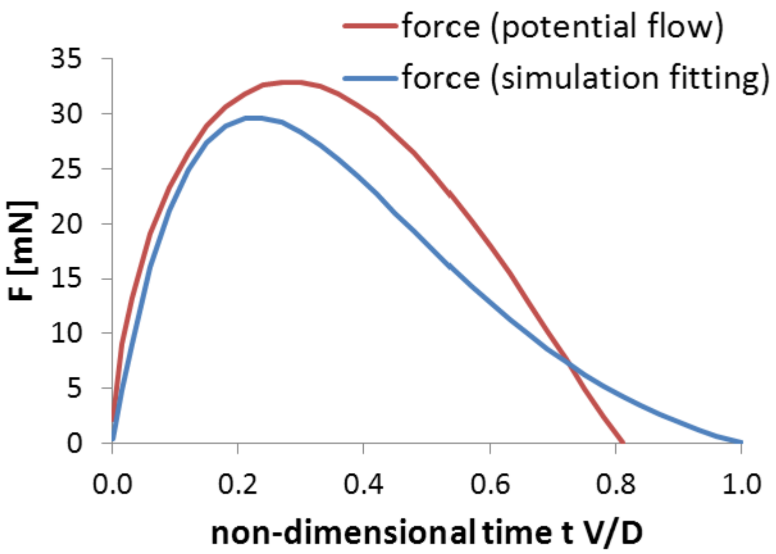

Figure S1: Drop impact force evolution in time for the impact of a millimetric drop with $D_0 = 2.8\text{mm}$ and $V = 2.86\text{m/s}$, with a convective time $t_{conv} = D/V = 1.01\text{ms}$. Forces were computed multiplying contact area (contact radius from Eq. 10) by pressure, obtained either with Eq. 8 (for potential flow theory) or with Eq. 9 (fitting results from numerical simulations [18]).

D. Table with drop deformation parameters at higher speeds

Table S1: Drop deformation parameters after impact on two tested substrates: maximum spread factor, $\xi_{\max}$, final spread factor, ξ_{final}, final contact angle, θ_{final}, and fluctuation time, t_f. Differences between soft substrate and hard substrates are provided.

V [m/s]	Surface	max spead ξ_{max}		max spread time $t_{\xi max}$ [ms]		final spread ξ_{final}		final contact angle θ_{final} [°]		oscillation time [ms]	
		data	soft/ hard	data	soft/ hard	data	soft/ hard	data	soft/ hard	data	soft/ hard
2.85	soft	4.2 ± 3%		2.9 ± 3%		5.35 ± 0%		31 ± 4		400 ± 28	
	hard	4.3 ± 0.4%	0.99	3.0 ± 3%	0.97	3.54 ± 2%	1.51	89 ± 2	0.34	745 ± 151	0.54
3.26	soft	4.4 ± 4%		2.5 ± 23%		5.33 ± 1%		28 ± 3		385 ± 38	
	hard	4.6 ± 0.3%	0.95	2.9 ± 6%	0.85	3.54 ± 3%	1.51	88 ± 1	0.32	742 ± 117	0.52
3.61	soft	4.4 ± 5%		2.6 ± 12%		5.49 ± 0%		22 ± 3		375 ± 12	
	hard	4.8 ± 1.0%	0.92	2.4 ± 34%	1.11	3.46 ± 2%	1.59	87 ± 3	0.25	725 ± 73	0.52

E. Sensitivity analysis for the Young modulus effect on surface deformation

A sensitivity analysis was performed to evaluate the effect of the Young modulus on the maximum deformation, $\delta_{\max}$. Results show that, assuming an "effective" Young modulus equal to $E_{eff} = 110kPa$ (i.e. approximately 6.5 times higher than the statically measured value of the Young modulus, by means AFM and deposited steel sphere), yield a very good agreement between $\delta_{\max}$ from numerical simulations (evaluated 1mm below the impact surface) and experimental results (see Figure S2a). In addition, Figure S2b shows the time evolution of substrate deformation (1mm below the impact surface) at different impact speeds. Figure S2b highlights that numerical simulation with modified Young modulus captures the time of maximum deformation correctly: $t_{\delta\max}$ ranges from 0.4 to 0.5ms, with very good agreement with experimental observations (~0.5ms). Differently, using the statically measured value of the Young modulus (17kPa), predicted $t_{\delta\max}$ ranges from 0.7 to 1ms. Differences of $t_{\delta\max}$ for different Young modulus arise because the propagation speed inside the substrate is a function of the Young modulus; although the wave propagation after drop impact is a

3D phenomenon (with axial symmetry), simulations show that the propagation speed is in the order of 1 m/s and close to the 1D longitudinal wave propagation speed, $V_{prop} = \sqrt{E/\rho}$ (equal to 4.2m/s for $E = 17kPa$, and 10.6m/s for $E = 110kPa$).

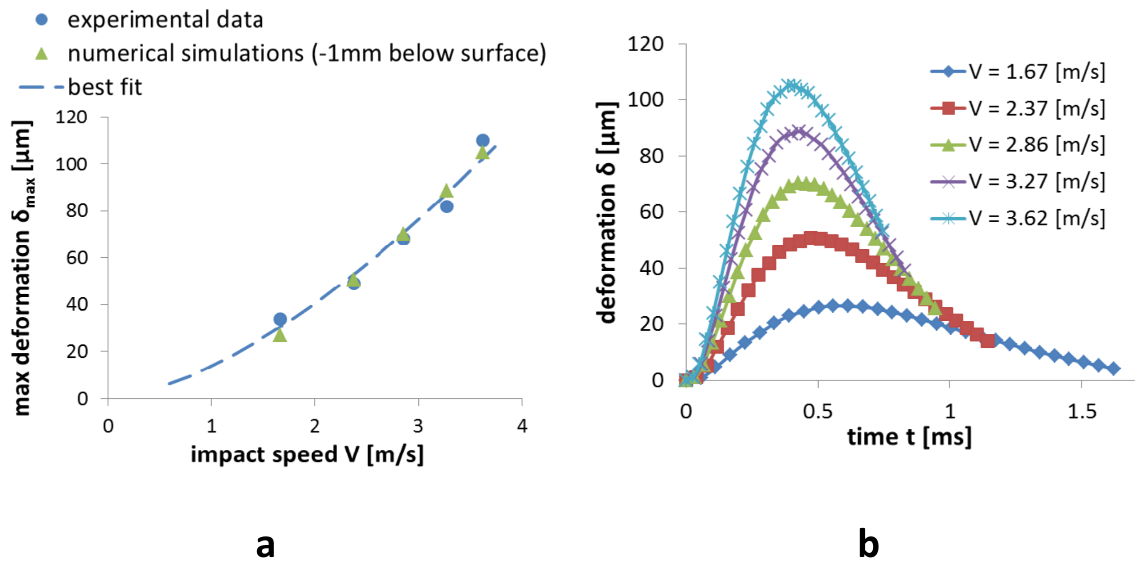

Figure S2: (a) Maximum deformation, $\delta_{\max}$, versus drop impact speed, V : comparison between experiments (circles) and numerical simulations using a modified Young modulus, $E_{app} = 110kPa$ (triangles). Dashed line represents the best fit for experimental data ($\delta_{\max} = 13.8V^{1.56}$). (b) Time evolution of substrate deformation, δ (evaluated 1 mm below the impact point, where the sensor was placed), resulting from numerical simulations using a modified Young modulus, $E_{app} = 110kPa$. Lines refer to different impact speed (see legend); impacting drop diameter is $D_0 = 2.8mm$.