Electronic Supplementary Information (ESI)

Beating synthetic cilia enhance heat transport in microfluidic channels

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Validation of thermal lattice Boltzmann model

We validated our thermal lattice Boltzmann model (TLBM) by performing four validation tests. The first test was of steady state heat conduction in a 2D domain. We simulated the temperature distribution in a square domain with the left, right and top walls heated at $\theta = 0$ and the bottom wall heated at $\theta = 1$, where θ is a dimensionless temperature. The results of the simulation were compared those obtained from the exact solution which is given by Eq. S1.[1]

$$\theta(x, y) = \frac{2}{\pi}\sum_{i=1}^{\infty}\frac{(-1)^{n+1}+1}{n}\sin\left[\frac{n\pi x}{L}\frac{\sinh(n\pi y/L)}{\sinh(n\pi)}\right] \qquad \text{(S1)}$$

Here, L is the length of the domain. A plot of the temperatures obtained from the TLBM simulations and the exact solution is provided in Fig. S1.

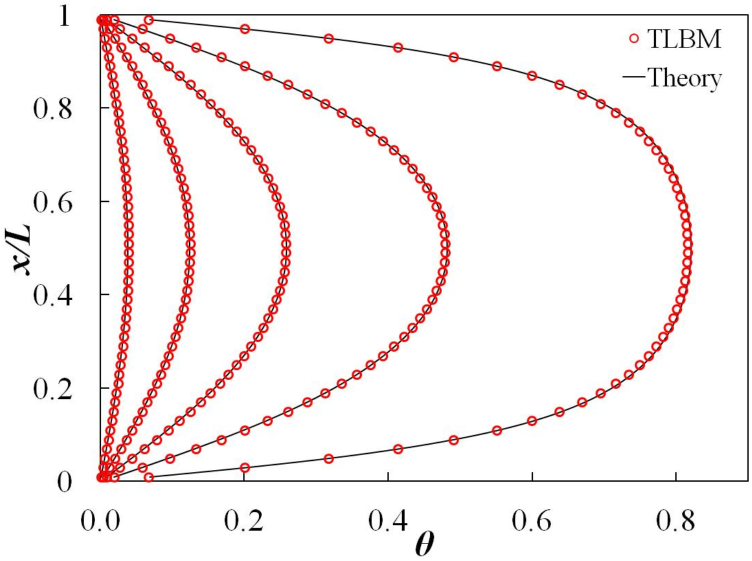

Figure S1. Temperature distributions in x direction at $y = 0.1L, 0.3L, 0.5L, 0.7L$ and $0.9L$ from simulation of steady state 2D heat conduction. Results from simulation (points) are compared with the exact solution (solid lines).

Our next validation test was performed by simulating 2-D transient conduction in a semi-infinite solid. The exact solution for this problem is provided by Eq. S2.[1]

$$\theta(x, y) = erf\left(\frac{x}{2\sqrt{\alpha t}}\right)erf\left(\frac{y}{2\sqrt{\alpha t}}\right) \qquad \text{(S2)}$$

Here, α is the thermal diffusivity and t is time. Plots of the isotherms for $\theta = 0.1, 0.3, 0.5$ and 0.9 at $t = 500\,\text{s}$ are provided in Fig. S2.

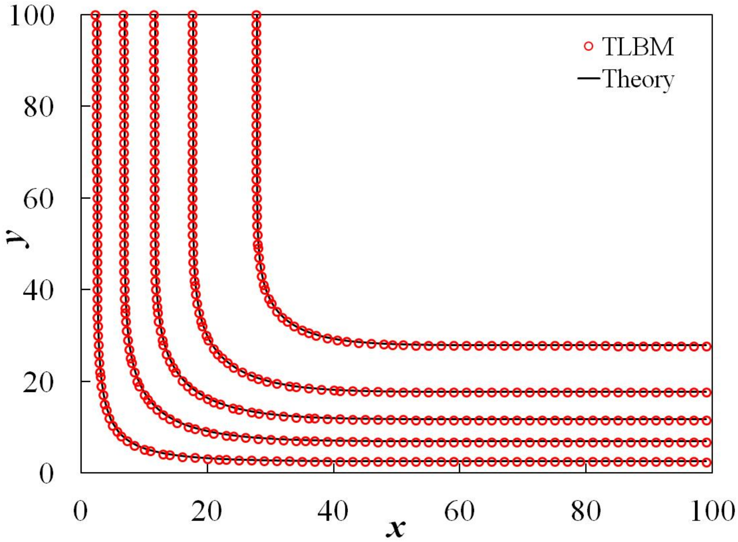

Figure S2. Location of isotherms for $\theta = 0.1, 0.3, 0.5$ and 0.9 at $t = 500\,\text{s}$ for transient 2D conduction in a semi-infinite solid. Results from simulation (points) are compared with the exact solution (solid lines).

The third validation test was a simulation of 1-D convection between two plates heated at $\theta = 1$ on the left and $\theta = 0$ on the left. The exact solution for this problem can be obtained from solving the ODE with corresponding boundary conditions shown in Eq. S3.[1]

$$\frac{\partial^2 \theta}{\partial x^2} - \frac{U}{\alpha}\frac{\partial \theta}{\partial x} = 0,\ \theta(x=0) = 1,\ \theta(x=L) = 0 \qquad \text{(S3)}$$

Simulations were performed for $U = 0.001, 0.005, 0.01, 0.05$ and 0.1 m/s. The results from this test are provided in Fig. S3.

The final validation test for on our model was performed by simulating 2D channel flow between two heated walls with a constant inlet temperature. The results from our model were compared to those obtained from a simulation performed using the commercial CFD package FLUENT. A plot of the normalized temperatures obtained from our model along with those obtained using FLUENT are provided in Fig. S4.

The validation simulations indicated that our TLBM accurately reproduces known analytical and numerical solutions for problems that involve conductive and convective heat transport.

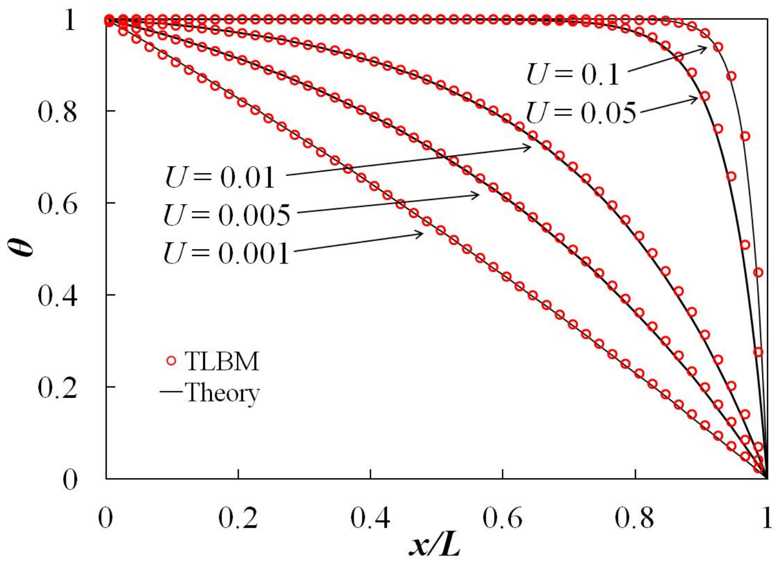

Figure S3. Temperature distribution in the x direction for several fluid velocities obtained from simulation of 1D steady state convection-diffusion between two heated plates. Results from simulation (circles) plotted against exact solution (solid lines).

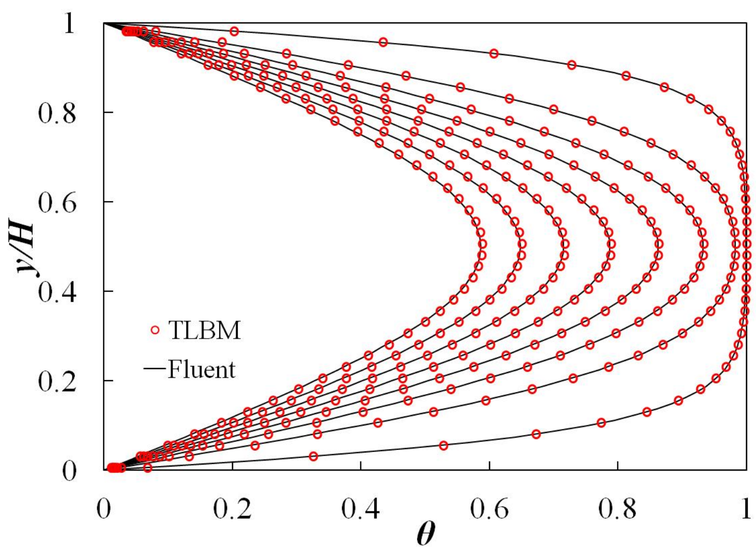

Figure S4. Temperature distributions in the y direction at distances of x/H = 0.07, 0.32, 0.57, 0.82, 1.07, 1.32, 1.57 and 1.82 from simulation of steady state convection in a channel. Results from TLBM simulation (points) are compared with those from FLUENT simulation (solid lines).

References

1. F. P. Incropera and D. P. DeWitt, *Fundamentals of heat and mass transfer*, 4th edn., Wiley, New York, 1996.