SUPPLEMENTARY INFORMATION

A. Supporting Online Movie

00:00:00 $\rightarrow$ 00:00:46 .-**Pushing Spiral** : A 50 μm film (bi-oriented polypropylene) is fixed to a rigid frame of size 0.8 m $\times$ 1.2 m, and an incision 7 mm in length is made that defines two cracks at points $\mathcal{A}$ and $\mathcal{B}$ (shown in the movie). With the help of a small cylinder of diameter 2 mm, we start pushing over one lip in the region closest to point $\mathcal{B}$. After the lip has rotated $3\pi/2$ radians the propagation of the spiral is fairly insensitive to the position of the tool.

00:00:46 $\rightarrow$ 00:01:31.- **Pulling Spiral** : A 50 μm film (bi-oriented polypropylene) is attached, at its edges, to a flat surface. At the center of it a circular hole is made and, tangentially to this hole, a notch is cut defining one crack point, $\mathcal{B}$ (shown in the movie). The end of the notch makes a convex hull with an exterior angle $\chi(\leq \beta)$. By pulling the resulting tear upwards, we form a fold which releases elastic energy by propagating a crack. The final crack path grows following a smooth logarithmic spiral shape.

B. Classic Fracture Mechanics: Determination of angles α and β.

A similar analysis to the one used by Hamm et al. [S1] and Audoly et al. [S2] is suitable to describe the fracture process leading to a spiral. We understand the process as the systematic repetition of the storage and release of energy, which can be separated into two steps:

1. Before rupture: The tool performs work by pushing the edge of the film from point $\mathcal{O}$ to point $\mathcal{P}$. Energy increases because of the stretching produced by the change in length of the original line $\overline{\mathcal{AOT}}$ (see Fig. S1(a)) into the new line $\overline{\mathcal{APT}}$.

2. Rupture: Fracture occurs for a fixed position of point $\mathcal{P}$. Thus, at some threshold value of the energy, it is more effective to release the elastic energy through fracture than to continue deforming. The release of energy is computed from the variation of the geometrical configuration when the crack propagates a length δs. Accordingly, point $\mathcal{T}$ is now $\mathcal{T}'$ and, therefore, the lengths d, L, and L', and the angles α and α' change to new values (see Fig. S1).

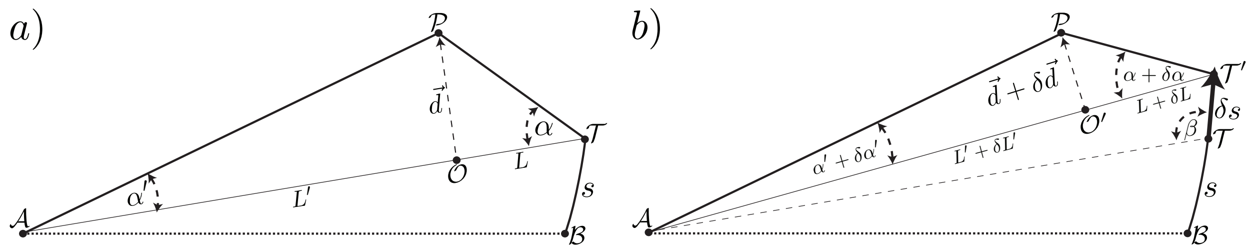

Fig. S 1. Variation of the geometry in the system owing to crack propagation. (a) Geometry before fracture. (b) Geometry after the crack moves a distance δs.

Having neglected bending energy, the total energy of the system is the sum of the surface energy $\gamma t s$ and the elastic energy associated with the deformation of line $\overline{\mathcal{AOT}}$ into line $\overline{\mathcal{APT}}$. The stretching of line $\overline{\mathcal{APT}}$ can be described by the distances d, L and L' defined in (see Fig. S1(a)). Thus, the total energy in the system is:

$$U = U_E(d, L, L') + \gamma t s \tag{S 1}$$

For a fixed position of point $\mathcal{P}$, the Griffith criterion applied to the last relation [S3] gives

$$\delta U = (\partial_d U_E)_{L,L'}\delta d + (\partial_L U_E)_{d,L'}\delta L + (\partial_{L'} U_E)_{d,L}\delta L' + \gamma t \delta s = 0$$

Note that the variation in the distance d is explained by the displacement of point $\mathcal{O}$ to the new point $\mathcal{O}'$ in Fig. S1(b). The total force applied is connected with the energy by the relation $\vec{F} = \partial_d U_E \hat{d}$, therefore, the magnitude of the force is given by $F = \partial_d U_E$ and the equilibrium equation can be simplified to

$$\delta U = F\delta d + \partial_L U_E \delta L + \partial_{L'} U_E \delta L' + \gamma t \delta s = 0 \tag{S 2}$$

By examining Fig. S1, it is straightforward to connect the variations of δd, δL, and $\delta L'$ to the change in fracture length δs. To first order, we obtain

$$\delta L' = \delta s \frac{L}{W} \tan\alpha \sin\beta \tag{S 3}$$

$$\delta L = -\delta s \cos\beta - \delta s \frac{L}{W} \tan\alpha \sin\beta \tag{S 4}$$

$$\delta d = -\delta s \frac{L'}{W} \sin\beta \tag{S 5}$$

where $W = L + L'$ is the unstretched length of the lip.

1. *First and Second Variations of the Energy*

Replacing Eq. (S 3), Eq. (S 4) and Eq. (S 5) into Eq. (S 2) gives for the first variation of the energy

$$\frac{\delta U}{\delta s} = -F\frac{L'}{W}\sin\beta - \partial_L U_E\left(\cos\beta + \frac{L}{W}\tan\alpha\sin\beta\right) + \partial_{L'}U_E\frac{L}{W}\tan\alpha\sin\beta + \gamma t = 0 \quad \text{(S 6)}$$

The constitutive relation for the force $F = F(d, L, L')$ allows the elimination of the parameter d in Eq. (S 6). Thus, an implicit relation for the force $F = F(\beta, L, L')$ is obtained. To find the direction of propagation, we require that the tear follows the direction where a minimal force is necessary to advance the crack, or equivalently, $(\partial_\beta F)|_{L,L'} = 0$. An implicit derivative of Eq. (S 6) gives the condition $\partial_\beta(\delta U/\delta s)$ that is usually referred to as the maximum-energy-release-rate criterion. Therefore, a second condition is obtained as

$$\frac{\partial}{\partial\beta}\frac{\delta U}{\delta s} = -F\frac{L'}{W}\cos\beta - \partial_L U_E\left(-\sin\beta + \frac{L}{W}\tan\alpha\cos\beta\right) + \partial_{L'}U_E\frac{L}{W}\tan\alpha\cos\beta = 0 \quad \text{(S 7)}$$

Solving Eqs. (S 6) and (S 7) for F and $\partial_L U_E$, we find the expressions

$$F = \frac{L}{L'}\tan\alpha\,(\partial_{L'}U_E - \partial_L U_E) + \frac{W}{L'}\gamma t\sin\beta \quad \text{(S 8)}$$

$$\partial_L U_E = \gamma t\cos\beta \quad \text{(S 9)}$$

2. *Energy and Force*

The elastic energy can be separated in two parts corresponding to the left and right ridges relative to the position of the tool. Hence,

$$U_E = U_l + U_r$$

From dimensional grounds, the stretching energies for the left and right ridges are $U_l = EtL'^2u(\alpha')$ and $U_r = EtL^2u(\alpha)$, respectively. In addition, $u(\alpha) \approx u(d/L) \approx a(d/L)^{n+1}$ for small angle α or $d/L \ll 1$, where a and n are dimensionless unknown positive numbers. It gives for the elastic energy

$$U_E = U_l + U_r = aEt\left[L'^2\left(\frac{d}{L'}\right)^{n+1} + L^2\left(\frac{d}{L}\right)^{n+1}\right] \quad \text{(S 10)}$$

We now can compute the total force. It yields

$$\begin{aligned} F &= \partial_d U_E \\ &= a(n+1)Et\left[L'\left(\frac{d}{L'}\right)^n + L\left(\frac{d}{L}\right)^n\right] \\ &= a(n+1)EtL\left[1+\left(\frac{L}{L'}\right)^{n-1}\right]\alpha^n \end{aligned} \tag{S 11}$$

Similarly, we explicitly compute the other remaining terms in Eqs. (S 8) and (S 9)

$$\begin{aligned} \partial_L U_E &= \partial_L U_r \\ &= -a(n-1)EtL\left(\frac{d}{L}\right)^{n+1} \\ &= -a(n-1)EtL\alpha^{n+1} \end{aligned} \tag{S 12}$$

$$\begin{aligned} \partial_{L'} U_E &= \partial_{L'} U_l \\ &= -a(n-1)EtL'\left(\frac{d}{L'}\right)^{n+1} \\ &= -a(n-1)EtL'\alpha'^{n+1} \end{aligned} \tag{S 13}$$

3. Determination of α and β

Equations (S 8) and Eq. (S 9) can be simplified to obtain the values of α and β. Replacing Eqs. (S 11), (S 12) and (S 13) into Eq. (S 8) gives the relation

$$a(n+1)EtL\alpha^n\left(1+\epsilon^{n-1}\right) = \gamma t\sin\beta\,(1+\epsilon) + a(n-1)EtL\alpha^n\epsilon\left(1-\epsilon^{n+1}\right) \tag{S 14}$$

where $\epsilon = L/L'$. Since $\alpha \ll 1$, the last term in Eq. (S 14) is of higher order in α and can be neglected. Thus, Eqs. (S 8) and (S 9) are

$$a(n+1)EtL\alpha^n\left(1+\epsilon^{n-1}\right) = \gamma t\sin\beta\,(1+\epsilon) \tag{S 15}$$

$$-a\,(n-1)\,EtL\alpha^{n+1} = \gamma t\cos\beta \tag{S 16}$$

We have two equations and two unknowns α and β. Solving for β, we obtain

$$\cot\beta = -\frac{(1+\epsilon)}{(1+\epsilon^{n-1})}\frac{n-1}{n+1}\alpha \tag{S 17}$$

There are an infinite number of solutions for β in the last expression, however, we know from our experiments that β must be close to $\pi/2$. Hence, $\cot\beta = \tan(\pi/2-\beta) \sim \pi/2-\beta$. Consequently, Eq. (S 17) gives

$$\beta = \frac{\pi}{2} + \frac{(1+\epsilon)}{(1+\epsilon^{n-1})}\frac{n-1}{n+1}\alpha \tag{S 18}$$

Using this relation in Eq. (S 15), we find for α

$$\alpha = \left[\frac{\ell}{aL(n+1)} \frac{1+\epsilon}{1+\epsilon^{n-1}} \right]^{1/n} \tag{S 19}$$

where $\ell = \gamma/E$. We now review the assumptions made to obtain Eqs. (2). Incidentally, Eqs. (2) are correct for the special cases $\epsilon = 0$ and $\epsilon = 1$, however, we can show that the region of validity is wider. The main requirement in our calculations is to have small angles $\alpha, \alpha' \ll 1$ along the fracture process. However, this cannot be true if the tool is too close to points $\mathcal{T}$ or $\mathcal{A}$ in Fig. S1. The divergence when the tool approaches point $\mathcal{T}$ is explicit from Eq. (S 19), however, it is a weak divergence due to the low exponent $1/n \approx 0.4$. The failure of our approach when the tool is close to point $\mathcal{A}$ is observed in Eq. (S 14) where a term of higher order in the angle α was neglected. Comparing this term to the left side of the same equation, we obtain the condition $(n+1)\left(1+\epsilon^{n-1}\right) \gg (n-1)\alpha^2\epsilon\left(1-\epsilon^{n+1}\right)$ for the validity of this approximation. Our experimental values $\alpha \sim 10^o$ and $n \approx 2.5$ show that this assumption is correct for $\epsilon \lesssim 4$ or $L \lesssim 0.8\,W$. Thus, the tool cannot be closer than a distance $W/5$ to point $\mathcal{A}$. Moving the tool outside this region, we conclude that α and β change no more than 2 and 0.9 degrees, respectively, due to variations in ϵ. Therefore, it is a good approximation to neglect ϵ in relations (S 18) and (S 19). It yields

$$\beta = \frac{\pi}{2} + \frac{n-1}{n+1}\alpha \tag{S 20}$$

$$\alpha = \left[\frac{\ell}{aL(n+1)} \right]^{1/n} \tag{S 21}$$

C. Logarithmic Spiral Characterization

The logarithmic spiral has a simple geometrical definition: the vector joining the center of the spiral to a given point $\mathcal{P}$ keeps a constant angle $\phi < \pi/2$ with the tangent to the spiral at the point $\mathcal{P}$ (see Fig. S2(a)). In polar coordinates the solution that satisfies this condition is $r = r_0 e^{\theta \cot\phi}$. Interestingly, our experiments suggest an alternative definition to a logarithmic spiral. The spiral growth of a tear is based on the construction of the convex hull by using the constant angle β between the tangents to the fracture trajectory at points $\mathcal{Q}$ and $\mathcal{P}$ (see Fig. S2(b)). This definition and the standard definition are, of course, equivalent. To show that, we will prove that there is a relation connecting the angle β with the spiral angle ϕ, which is the same for any given position of point $\mathcal{P}$ in the spiral.

Electronic Supplementary Material (ESI) for Soft Matter

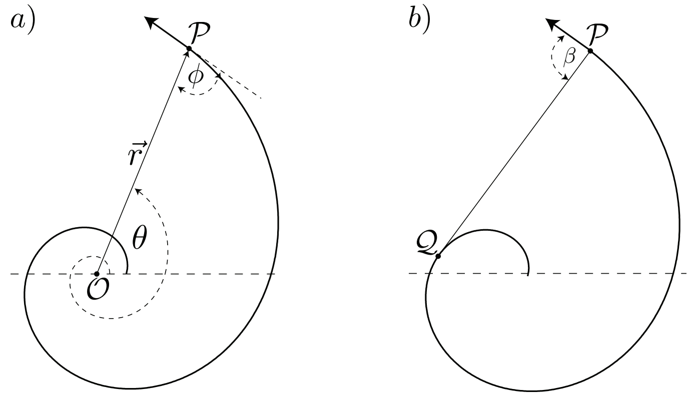

Fig. S 2. The logarithmic spiral (a) and the experimental spiral (b).

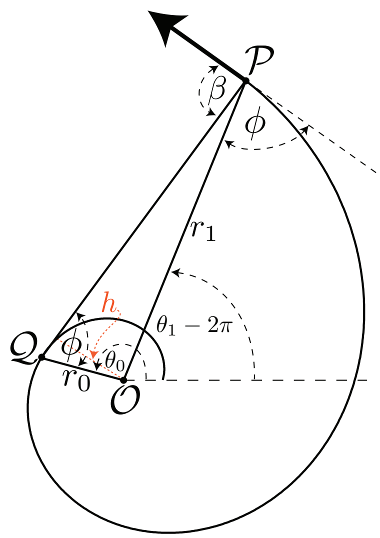

Fig. S 3. Geometry in the logarithmic spiral

The proof comes from simple geometry. Points $\mathcal{Q}$ and $\mathcal{P}$ have the polar coordinates (r_0, θ_0) and (r_1, θ_1), respectively (see Fig. S3). If we set $\theta = 0$ to coincide with the horizontal axis, line $\overline{\mathcal{OP}}$ has an angle $\theta_1 - 2\pi$ with this axis. Geometry in the triangle $\mathcal{QOP}$ shows that the angle $\angle\mathcal{QOP} = \beta$ (see Fig. S3), hence we have the condition $\theta_0 - (\theta_1 - 2\pi) = \beta$.

The height h of triangle $\mathcal{POQ}$ in Fig. S3 can be written in two equivalent ways as

$$h = r_0 \sin\phi = r_1 \sin(\pi - \beta - \phi)$$

which gives the relation

$$\frac{r_0}{r_1} = \frac{\sin(\beta + \phi)}{\sin\phi} \qquad \text{(S 22)}$$

Since points $\mathcal{Q}$ and $\mathcal{P}$ belong to the logarithmic spiral, we obtain $r_0 = e^{\theta_0 \cot\phi}$ and $r_1 = e^{\theta_1 \cot\phi}$. Thus,

$$\frac{r_0}{r_1} = e^{(\theta_0-\theta_1)\cot\phi} = e^{(\beta-2\pi)\cot\phi} \qquad \text{(S 23)}$$

Replacing Eq. (S 23) into Eq. (S 22), we finally obtain the transcendental relation connecting β and ϕ

$$\sin\phi \; e^{-(2\pi-\beta)\cot\phi} - \sin(\beta+\phi) = 0 \qquad \text{(S 24)}$$

[S1] E. Hamm, P. Reis, M. Leblanc, B. Roman, E. Cerda, *"Tearing as a test for mechanical characterization of thin adhesive films"*, Nature Materials, **7** , 386 - 390 (2008).

[S2] B. Audoly, P.M. Reis and B.Roman, *"Cracks in thin sheets : When geometry rules the fracture path"*, Phys. Rev. Lett.**95** , 025502 (2005).

[S3] A. A. Griffith, *"The Phenomena of Rupture and Flow in Solids"*, Phil. Trans. of the Roy. Soc. of London, A, **221** , 163-198 (1921).