1 Supplementary information

The equations and assumptions used to derive the hinge torque model are presented below. Representative values for physical constants are listed in Table 1. These values are for a hinge with a 22 mil thick SMP layer, a six millimeter wide resistive circuit, and supplied with two amps of current.

Variable		value	unit
Hinge Length	L	30	mm
Trace width	w_{trc}	6	mm
Bond thickness	t_{bond}	127	μm
Resistance	R_o	0.59	Ω
conductivity	k	0.2	W/mK
diffusivity	α	$7.97 \cdot 10^{-8}$	m^2/s
transfer coefficient	h_{smp}	30.8	W/m^{2}K
Bending Stiffness	I_b	0.037	Nm/rad
SMP Thickness	t_{smp}	560	μm
Surface Coefficient	C_s	0.92	
Young's Modulus	E	400	kPa
Specific energy	q_{gen}	8.7	kW/m^2
Transition temperature	T_g	95	°C
Ambient temperature	T_o	25	°C

Table 1 Known and calculated values used in the thermal and mechanical models of a test hinge with a 22 mil PO layer, six millimeter wide trace, and supplied with two amps of current

1.1 Convection

Convection is the SMP heat transfer coefficient h_{smp} multiplied by the difference between surface temperature and ambient temperature. It can be expressed by the following:

$$h_{smp} = \frac{Nuk_{air}}{L_c} \tag{1}$$

In this case, the Nusselt number $Nu = 3.1$[1], the thermal conduction of air $k_{air} = 0.026W/mK$[2], and the characteristic length L_c depends on the trace geometry.

$$L_c = \frac{Lw_{trc}}{2(L + w_{trc})} \tag{2}$$

1.2 Y-axis and vertical heat transfer

The geometry of the hinge is effectively constant in the y-direction, so we also assume that the hinge is isothermal in that direction. In the z-direction (vertical), the Biot number $Bi = h_{smp}t_{smp}/k_{smp}$ is much smaller than one for all hinges tested, indicating that conduction through the SMP will occur more quickly than convection. Because of this, we can ignore the transient effects of vertical conduction, and assume that the vertical heat flux through the SMP Q_{cond} is equal to the heat flux due to convection Q_{conv} from the SMP surface.

$$Q_{conv} = Q_{cond} \tag{3}$$

$$h_{smp}T_{surface} = \frac{\left(T - T_{surface}\right)k}{t_{smp}} \tag{4}$$

$$T_{surface} = C_sT = \frac{k}{k + h_{smp}t_{smp}}T \tag{5}$$

In this way we can solve for the ratio C_s of the surface SMP temperature T_s to the inner SMP temperature T. For all of our experimental hinges, $C_s > 0.9$. Therefore, for the sake of simplification, we assume the material is isothermal in the y- and z-directions, and can be treated as a one-dimensional system. We still use C_s when comparing our model to experimental thermal data, because the SMP temperature is measured at the surface using thermal imaging.

1.3 Horizontal heat transfer

Using the similarity solution for the transient, semi-infinite heat transfer problem[2], we find that

$$\frac{T - T(0,t)}{T_o - T(0,t)} = \text{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right) \tag{6}$$

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = q_{gen} - \frac{Q_{conv}}{A} \tag{7}$$

$$= q_{gen} - h'\left(T(0,t) - T_o\right) \tag{8}$$

$$\frac{T(x,t) - T_o}{q_{gen}/h'} = \left[\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \right. \tag{9}$$

$$\left. \exp\left(\frac{h'x}{k} + \frac{h'^2\alpha t}{k^2}\right)\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h'\sqrt{\alpha t}}{k}\right)\right] \tag{10}$$

The specific power generated q_{gen} is considered to be the total power released by the resistive circuit Q_{gen} over the total area through which it is diffusing. We treat the core as a box out of which heat conducts uniformly.

$$A = 2(t_{smp} + w_{trc})L \tag{11}$$

$$q_{gen} = \frac{Q_{gen}}{A} \tag{12}$$

Q_{gen} is a function of the resistance of the circuit and the current running through it. The resistance is itself affected by the temperature; from observation, we conclude that the temperature difference in degrees Celsius is approximately equal to the power dissipated by the circuit per unit length in watts per meter. This relationship is modeled below

$$Q_{gen} = I^2 R_o (1 + \Delta T_{est} \alpha_{cu}) \tag{13}$$
$$\approx \Delta T_{est} L \tag{14}$$
$$\approx \frac{I^2 R_o L}{L - \alpha I^2 R_o} \tag{15}$$

h' is the lumped heat transfer coefficient including convection from both the core (at $x = 0$) and the margin. First, we must find the relationship between h', governing heat transfer across A, to the convective heat transfer coefficient of the SMP h_{smp} occurring at the surface of area A_s.

$$\Delta T = (T(0,t) - T_o) \tag{16}$$
$$Q_{conv} = h_{smp} A_s \Delta T = h' A \Delta T \tag{17}$$

A_s accounts for both the core and marginal area undergoing convection. The marginal convection is a function of the marginal temperature and the effective distance over which convection is occurring. We approximate the temperature as linearly decreasing away from the core; therefore, the average marginal temperature is assumed to be half of the core temperature, and the distance is approximated from the similarity solution.

$$Q_{core-conv} = h_{smp} \Delta T L w_{trc} \tag{18}$$
$$Q_{margin-conv} = h_{smp} \frac{\Delta T}{2} L w_{margin} \tag{19}$$
$$w_{margin} \approx 2\sqrt{\alpha t} \tag{20}$$
$$Q_{conv} = Q_{core-conv} + Q_{margin-conv} \tag{21}$$
$$A_s = \frac{Q_{conv}}{h_{smp} \Delta T} = L\left(w_{trc} + \sqrt{\alpha t}\right) \tag{22}$$
$$h' = \frac{h_{smp} w_{trc}}{2(t_{smp} + w_{trc})} \left(1 + \frac{\sqrt{\alpha t}}{w_{trc}}\right) \tag{23}$$

In addition to finding the core temperature, these equations are used to find the distance from the core in which the SMP has been activated. The first order Taylor approximation of T is found about $x = 0$, and is rewritten as follows:

$$x_a = \left[1 - \frac{T_g h'(k + h' t_{smp})}{q_{gen} k} - \exp\left(\frac{h'^2 \alpha t}{k^2}\right) \text{erfc}\left(\frac{h'\sqrt{\alpha t}}{k}\right)\right] \cdot \left[\frac{k}{h' \exp\left(\frac{h'^2 \alpha t}{k^2}\right) \text{erfc}\left(\frac{h'\sqrt{\alpha t}}{k}\right)}\right] \tag{24}$$

1.4 Torque

The steady state torque is determined by integrating the stress σ along the midplane.

$$\tau_s = \int_{t_{bond}}^{t_{smp}+t_{bond}} \sigma x L \, dx \tag{25}$$
$$= \frac{\left(t_{smp}^2 + 2 t_{bond} t_{smp}\right) \sigma L}{2} \tag{26}$$

Assuming plane strain, we replace:

$$\sigma = \frac{E\varepsilon}{1 - \nu^2} \tag{27}$$
$$= \begin{cases} 0 & T(0,t) < T_g - 15 \\ \frac{4E}{3}\left(\frac{T(0,t) - T_g + 15}{30}\right) & T_g - 15 < T(0,t) < T_g + 15 \\ \frac{4E}{3} & T(0,t) > T_g + 15 \end{cases} \tag{28}$$

where the Poisson's ratio $\nu = 0.5$.

The increase in torque due to deformation is considered to be a separate model that assumes the SMP layer to be a one-dimensional element of length w_{act}, under tension equal to σ times the cross sectional area, fixed to each face. The deformed face is at an angle ϕ with the ground close to the hinge (Fig. 3 C). τ_d is proportional to the force from the tension element perpendicular to the face F_{perp} and the distance from the hinge on which the force is acting r.

$$\tau_d = F_{perp} r \tag{29}$$

In this model, the tension element forms a triangle with both faces of the fold extending a length r from the hinge. The angle between each face and the tension element is $\phi/2$. We assume ϕ is small, in order to simplify the trigonometry.

$$w_{act}/2 = r\cos(\phi/2) \tag{30}$$
$$r \approx w_{act}/2 \tag{31}$$
$$F_{perp} = F_{tension} \sin\left(\frac{\phi}{2}\right) \approx (L t_{smp} \sigma) \frac{\phi}{2} \tag{32}$$
$$\tau_d \approx (L t_{smp}) \sigma \frac{w_{act} \phi}{4} \tag{33}$$

We then model the deformed face as a bending beam, and assume that $\tau = I_b \phi$, where I_b is the bending stiffness.

$$\tau = \tau_s + \tau_d \tag{34}$$
$$\approx \frac{2ELI_b\left(t_{smp}^2 + 2 t_{bond} t_{smp}\right)}{3I_b - EL t_{smp} w_{act}} \tag{35}$$

1.5 Critical current

From simulations, it is apparent that after some amount of time the growth of the core temperature begins to slow. In order to develop guidelines for picking an appropriate current, we invent a critical time point t^* indicating when the temperature growth slows. Because erfc dominates the equation, and erfc(χ) approaches zero at $\chi \approx 1$, we choose this as our critical point, and attempt to find a recommended current I_r that achieves $T(0,t^*) = T_g$ at this point.

$$\chi = \frac{h'\sqrt{\alpha t}}{k} = 1 \tag{36}$$

$$T(0,t) - T_o = \frac{q_{gen}}{h'}\left[1 - \exp(\chi)\,\mathrm{erfc}(\chi)\right] \tag{37}$$

$$= 0.57\frac{q_{gen}}{h'} \tag{38}$$

By using eq. 23, and assuming $t_{smp} << w_{trc}$, we can approximate

$$h' \approx \frac{h_{smp}}{2}\left(1 + \frac{k}{w_{trc}h'}\right) \tag{39}$$

$$\approx \frac{h_{smp}}{2}\left(1 + \frac{k}{w_{trc}h_{smp}}\right) \tag{40}$$

$$q_{gen} \approx \frac{I^2 R}{2Lw} \tag{41}$$

$$T(0,t) - T_o \approx \frac{0.57 I^2 R}{L(h_{smp}w + k)} \tag{42}$$

$$I_r = \sqrt{\frac{(T_g - T_o)(h_{smp}w + k)L}{0.57R}} \tag{43}$$

References

1 I. Martorell, J. Herrero and F. X. Grau, *International Journal of heat and mass transfer*, 2003, **46**, 2389–2402.

2 D. P. Dewitt and F. P. Incropera, *Fundamentals of heat and mass transfer*, Wiley, 2002.