Electronic Supplementary Material (ESI) for Soft Matter


## SI1

In order to demonstrate the HES mechanism and its products we manufacture a simple elastic model: Two planar latex sheets are stretched uniaxially along perpendicular directions and then glued one on top of the other. After release from external constrains, the compound sheet is defined to have a local saddle-like target curvature, due to length differences along the thin axis. The sheet is residually stressed and will adopt different configurations upon cutting into different shapes. A small enough circle will adopt a saddle shape. Large enough sections will become envelopes of a cylinder.

Helical configurations are achieved when cutting strips from this sheet along a direction that forms an angle $\theta$ with one of the stretching directions. The result helical configuration will depend on the cutting angle $\theta$, on the strip width $w$, and on the amount of stretching i.e. amount of curvature $k_0$.

More information can be found in (Armon, et al. 2011).

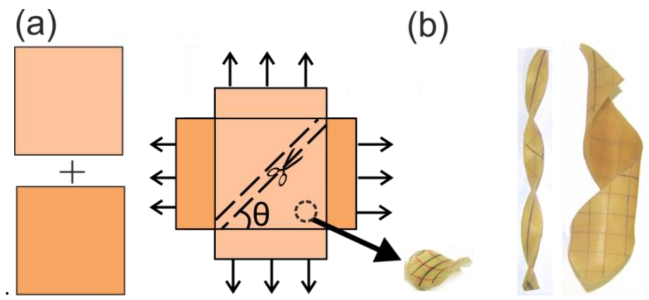


**Fig 1**: (a) Preparation of a bilayer latex sheet. (b) Examples of equilibrium shapes of different cut sections from the sheet. A small disk adopts a saddle configuration. A narrow enough strip cut in 45° adopts a twisted configuration. A wide enough strip adopts a helical one.

## SI2

A strip can relax both its in-plane strain and first strain derivatives along its mid-line using in-plane deformations only, thus leaving the curvatures intact and not contributing to the bending energy. This is not true for second strain derivatives, since determining the second metric derivatives imposes a certain Gaussian curvature according to the Gauss-Codazzi-Mainardi equations. We will therefore have, to leading order in $w$, strain of the form $a - \bar{a} \propto (K - K_{\bar{a}})u^2$, where $K$ is the Gaussian curvature, $K_{\bar{a}}$ is the reference Gaussian curvature dictated by $\bar{a}$, and $u$ is the in-plane distance from the mid-line. The leading contribution to the stretching energy, which is quadratic in the strain tensor, will therefore take the form

$$E_s \propto tw^5(K - K_{\bar{a}})^2.$$

As for the bending energy, there is no reason for the leading terms to vanish (this may contribute to the stretching energy). The leading term will therefore (by integrating the full local expression over the width) take the form

$$E_b \propto t^3 w \left\| b - \bar{b} \right\|^2$$

where $\left\| b - \bar{b} \right\|^2 \equiv \left[ (1-\nu) Tr\left[ \left( b - \bar{b} \right)^2 \right] + \nu Tr^2 \left( b - \bar{b} \right) \right]_{midline}$.

**SI3**

The preferred curvature $k_0$ and Poisson's ratio $\nu$ are needed in order to calibrate our model. As these are not generally known for macro-molecular membranes, we shall now provide an easy method for inferring them from measurements.

The pitch of the twisted configuration in the narrow limit is predicted to take the form

$$Pitch_{narrow} = \frac{2\pi \sin(2\theta)}{k_0}.$$

The pitch of the helical configuration in the wide limit is predicted to take the form

$$Pitch_{wide} = \frac{2\pi \tan(\theta)}{k_0(1-\nu)}.$$

(the tangent can be replaced with a cotangent for the pitch of the meta-stable state in that limit). The ratio between those measurable pitches gives

$$\frac{Pitch_{narrow}}{Pitch_{wide}} = 2(1-\nu)\cos^2(\theta)$$

For the specific common case of $\theta = 45°$, one easily extracts the Poisson's ratio:

$$\nu = 1 - \frac{Pitch_{narrow}}{Pitch_{wide}}$$

The radius of the helical configuration in the wide limit, and hence that of the closed tube, is predicted to take the form:

$$Radius_{tube} = \frac{1}{k_0(1-\nu)},$$

and so we also obtain the preferred curvature:

$$k_0(1-\nu) = Radius_{tube}^{-1}.$$

**SI 4**

We use a 2D finite element simulation, which models the ribbon as a 2D triangular mesh. To every triangle we attribute the intrinsic metric tensor $\bar{a}$ (in the HES model the unity tensor)

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and intrinsic curvature tensor $\bar{b}$ (in the HES model $\begin{pmatrix} k_0\cos(2\theta) & -k_0\sin(2\theta) \\ -k_0\sin(2\theta) & -k_0\cos(2\theta) \end{pmatrix}$). We also provide the elastic parameters $Y, \nu$ and the thickness $t$.

A spatial configuration of the mesh is given in the form of the 3D positions of its vertices. The metric tensor $a$ and curvature tensor $b$ of a configuration are assessed locally for each triangle from the conformation of itself and its near neighbors. These are then plugged into the energy functional [eq. in article] together with the intrinsic parameters and summed over all triangles in order to obtain the elastic energy of the sheet.

The sheet is given some initial condition, from which minimization of the elastic energy with respect to vertex coordinates is carried out using a steepest descent method. The simulation stops at a local minimum of the energy, which represents an equilibrium state. The attributes of that state (pitch, radius, curvatures etc.) are then measured externally.

Running simulations with various parameters allows us inspection of the equilibrium configurations for the entire phase space of $\theta, k_0, \nu$ and $t$ (hence also $\tilde{w}$). Meta-stable states are obtained by starting the simulation from an appropriate initial condition (the analytic expression of that state in the wide limit).

<u>**SI5**</u>

**Movie#1 - gel model**: A strip cut in $\theta = 45^o$ is being heated in water and undergoes the transition from twisted to helical ribbon configuration and closes into a tube.

The movie starts as the water temperature is lower than polymerization temperature. As a result the gel is swollen compared to the cotton strings, which creates curvatures. As the temperature is increased, the gel shrinks and the target curvatures change sign, i.e. the handedness is changed from left to right. Field of view: $13 \times 4$ cm. Real time of the procedure: 2 hours. Temperature interval: 30-40$^o$C.

**Movie #2 – simulation**: equilibrium configurations of an HES strip at $\theta = 45^o$ as calculated by the elastic simulation, and the corresponding $\tilde{w}$ values. Along the simulation, the thickness of the strip is being reduced, which is equivalent to any other change of parameters that increases $\widetilde{w}$, like increase in w or in $k_0$. The color represents bending energy density.