Buckling dynamics of a solvent-stimulated stretched elastomeric sheet

Supplemental materials

Alessandro Lucantonio,[1,*] Matthieu Roché,[2,†] Paola Nardinocchi,[1] and Howard A. Stone[2]

[1] *Dipartimento di Ingegneria Strutturale e Geotecnica, Sapienza Università di Roma, Rome, Italy*

[2] *Department of Mechanical and Aerospace Engineering, Princeton University, Princeton NJ, USA*

(Dated: January 13, 2014)

* Now at: SISSA, International School for Advanced Studies, via Bonomea 265, 34136 Trieste, Italy; E-mail address: alessandro.lucantonio@sissa.it

† Now at: Laboratoire de Physique des Solides, Université Paris Sud - CNRS UMR 8502, Bâtiment 510, 91405 Orsay, France.

I. WAVELET TRANSFORM PROFILOMETRY

A fringe pattern with frequency f_0 and sinusoidal intensity profile was projected on the sheet that, in its flat configuration, lied in the reference plane xy (Fig. 1), with the x and y axes spanning the longitudinal and transverse directions of the sheet, respectively. A camera forming an angle $\theta = 45$ deg with the reference plane took snapshots of the sheet during the experiment. First, images were processed through a MATLAB script to filter out noise: this was achieved by computing, for each line (x fixed) of the picture, the Morley wavelet transform of the signal $s(x,y)$ and extracting only its fundamental component, that is the frequency content of the signal near f_0. Then, the phase difference $\Delta\Phi(x,y)$ between the fundamental component of the deformed fringe pattern and the fundamental component of the reference fringe pattern was computed. This phase difference is related to the vertical displacement $h(x,y)$ of the sheet through the geometrical features of the setup.

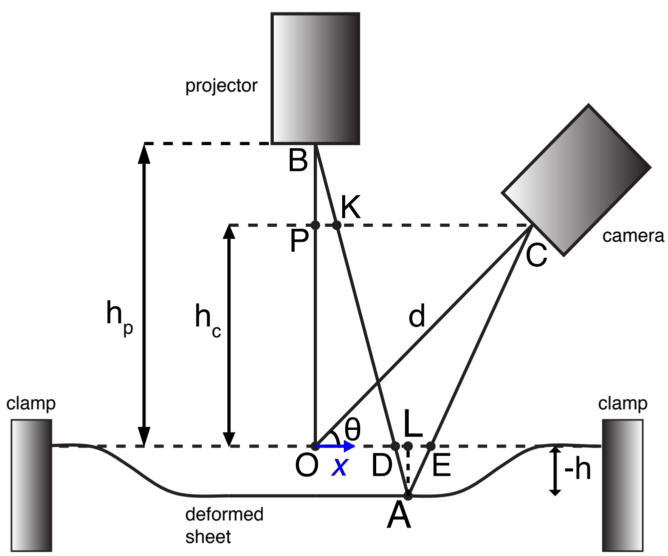

FIG. 1. Optical geometry of the experimental setup.

To establish the relation between the phase shift of the fringe pattern and the height of the deformed sheet above the reference plane, let us first define the following geometrical quantities:

$$\Delta h = |h_p - h_c|, \qquad x_p = \overline{OL}, \qquad d = \overline{OC}, \tag{1}$$

where h_p and h_c are the heights of the projector and the camera, respectively, with respect to the reference plane (Fig. 1). Noting that the triangles ADE and AKC are similar, we can write:

$$\frac{\overline{DE}}{\overline{KC}} = \frac{-h}{-h+h_c} \longrightarrow h = \frac{h_c \, \Delta\Phi}{\Delta\Phi - 2\pi f_0 \, \overline{KC}}, \tag{2}$$

with $\Delta\Phi = 2\pi f_0 \overline{DE}$ the phase difference computed through the wavelet transform analysis. Moreover, from Fig. 1,

$$\overline{KC} = d\cos\theta - \overline{PK} = d\cos\theta - \frac{\Delta h}{h_p - h} x_p \, . \tag{3}$$

Inserting (3) into (2) and rearranging the terms, we obtain a second order equation for h:

$$A\,h^2 + B\,h + C = 0, \tag{4}$$

with

$$A = 2\pi f_0 \, d\cos\theta - \Delta\Phi, \tag{5}$$

$$B = -Ah_p + 2\pi f_0 \, \Delta h \, x_p + h_c \Delta\Phi, \tag{6}$$

$$C = -h_c h_p \Delta\Phi \, . \tag{7}$$

The correct solution of (4) is selected through the condition $\Delta\Phi < 0 \implies h > 0$.

II. EFFECT OF THE INITIAL ASPECT RATIO ON THE WAVELENGTH

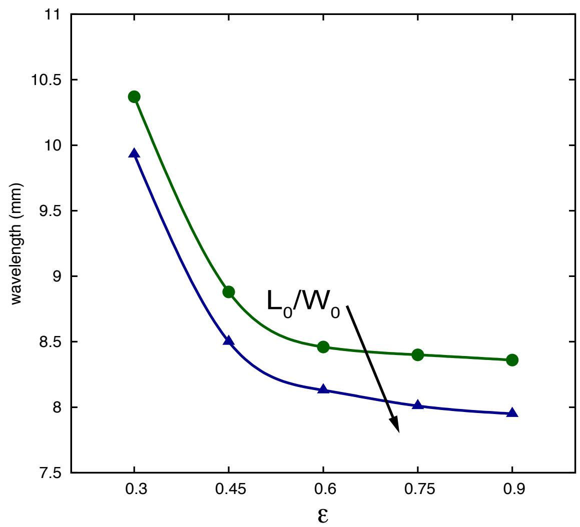

FIG. 2. Wavelength at the end of the swelling transient ($t = 600$ s) as a function of the applied strain computed from numerical simulations for two different values of the initial aspect ratio L_0/W_0: 5/6 (triangles), 5/8 (circles). The value of the wavelength (~ 8.9 mm) at $\varepsilon = 0.45$ for the sheet with $L_0/W_0 = 5/8$ compares well with the value of ~ 9.4 mm we measured experimentally. Overall, by decreasing the aspect ratio of 25 % from $5/6$ to $5/8$, the maximum increase in wavelength in the range $0.3 \leq \varepsilon \leq 0.9$ is about 6 %.

Electronic Supplementary Material (ESI) for Soft Matter

III. DYNAMICS OF WAVELENGTH AND AMPLITUDE OF THE WRINKLES

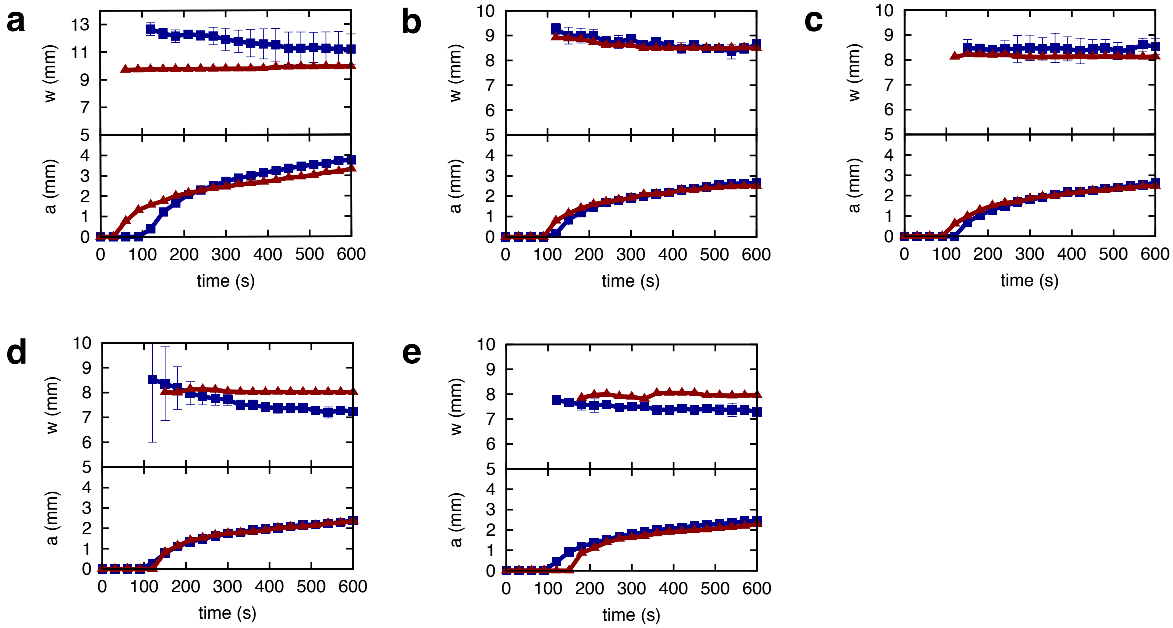

FIG. 3. Dynamics of the wavelength w and amplitude a of the wrinkles for different initial strains ε: (a) 0.3, (b) 0.45, (c) 0.6, (d) 0.75, (e) 0.9. Squares: average experimental values along the width at the center of the sheet; error bars are the standard deviations of the measurements along the width. Triangles: numerical simulations (see the main text of the paper for information on the model). While amplitude grows with time following the increase of solvent mass uptake, the wavelength is nearly time-independent. The slight decrease of the wavelength is a consequence of the relaxation of the edges (see discussion in the main text of the paper and Movie S1).

A. Movies

Movie S1: sequence of frames of the Shore 35A sheet taken every 30 s during the swelling transient, for $\varepsilon = 0.9$. At time $t = 0$ s the bottom surface of the elastomer is brought into contact with the solvent bath. The swelling of the sheet with solvent causes a lateral expansion accompanied by a rapid curling upwards of the free edges. Then, the sheet becomes mechanically unstable and a time-evolving wrinkling pattern develops across the width of the wetted sheet. As the absorption of the solvent proceeds, the amplitude of the wrinkles grows, while the edges relax (the local curvature decreases) and bend downwards. The relaxation of the edges is accompanied by a decrease of the average wavelength (see also Fig. 3 Supplemental Material), because new wrinkles forming at the edges drive the others toward the center.