Supplemental Material for

"A universal scaling law of grain chains elasticity under pressure revealed by a simple force vibration method"

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The supplemental material includes four parts. The first is a comparison of the resonant frequency of the system unloaded and loaded with fine sand. The second part shows some additional information of the multiple resonant dissipation peaks of grain chains with different lengths in sand. The third displays the dissipation behavior of glass beads with smooth and spherical surface. The fourth displays the shear amplitude dependence of dissipation peaks in sand.

I. Resonant frequency of the system unloaded and loaded with fine sand

In Fig. S1, the energy dissipation of the system unloaded and loaded with fine sand is plotted as a function of frequency over a range of 0.3-200 Hz. It can be seen that without granular material, the system resonant frequency is 67.5 Hz. In the presence of fine sand the pendulum resonance frequency f_r = 139.6 Hz is much higher than that without sand. Furthermore, the inset figure shows that in the studied frequency range of 0.5-9.0 Hz, the system loaded fine sand exhibits typical dissipation peaks while the unloaded system shows no change. This gives a further evidence that the loss peaks are related to the dissipation behaviors of granular materials rather than the resonance of the whole system.

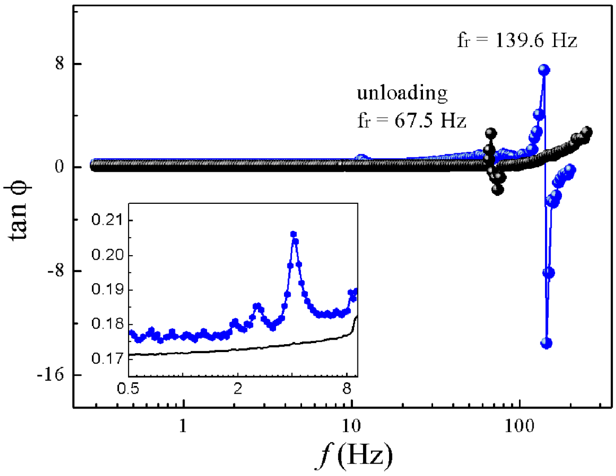

Fig. S1. Frequency dependence of energy dissipation of the system unloaded and loaded with fine sand. The inset shows the comparison results of energy dissipation for the two systems over a frequency range of 0.5-9.0 Hz.

II. Multiple resonant peaks of grain chains with different lengths in sand

Figure S2 shows typical loss spectra (tanϕ and G) of fine sand with shear amplitude $A_0 = 7.84 \times 10^{-6}$ m. It can be clearly seen that loss tangent exhibits P1, P2, and P3 peaks (solid lines) and at the same time modulus show three minima, corresponding to resonance dissipation of grain chains with two, three, and four particles, respectively. In addition, at the low frequency side, more small tanϕ peaks and G minima are vaguely observed, labeled as P4 and P5 (dotted lines), which should be associated with the dissipation of longer grain chains. Note that the dissipation behavior of longer grain chains seem be to be reflected more apparently by modulus than loss tangent.

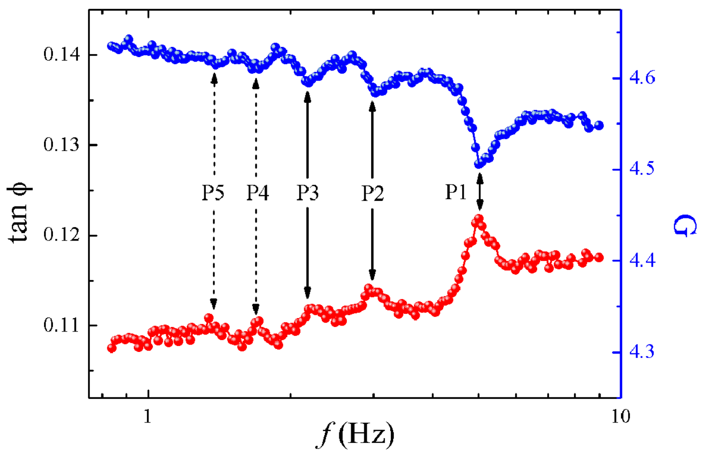

Fig. S2. Mechanical spectra (energy dissipation and modulus) of fine sand as a function of frequency at the amplitude $A_0 = 7.84 \times 10^{-6}$ m.

III. Dissipation behavior of glass beads with smooth and spherical surface

As mentioned in the main text, the effect of surface roughness of the particles on the dissipation behavior is considered. The surface roughness of the particles was measured by an optical microscopy, as shown in Fig. S3. It can be seen that the surface of glass beads is smooth and spherical, while that of sand is rough and non-spherical in shape. Figure S4 provides loss tangent results of glass beads with smooth surface but the same particle diameter to fine sand. The energy dissipation exhibits similar loss peaks P1, P2, and P3 with decreasing frequency and these peaks are lower compared to those of sand. All the peaks shift to high frequency with increasing pressure, but the ratios of peaks positions for P1, P2, and P3 peaks $f_1/f_2 \approx \sqrt{3}$ and $f_1/f_3 \approx \sqrt{6}$, invariable with increasing pressure. In addition, this result also suggests that the dissipation peaks are not caused by grain slip or any interaction between the grains and the probe. If not, these peaks of glass beads should be higher and sharper compared to those of sand. Thus, the dissipation peaks are associated with the inherent resonance of grain chains with different lengths.

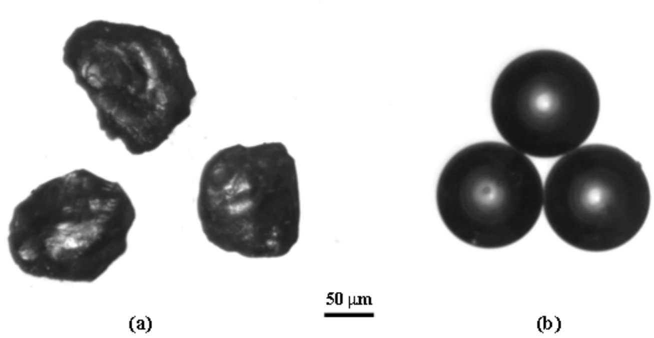

Fig. S3. Optical microscopy images of (a) fine sand (b) glass beads.

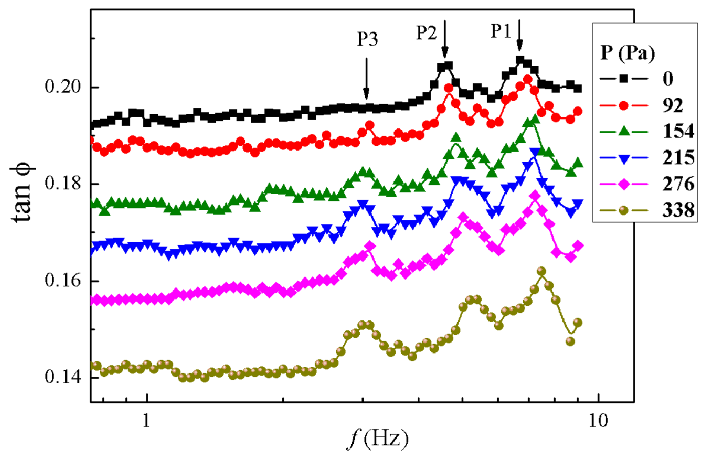

Fig. S4. Energy dissipation of glass beads with polish surface as a function of pressure from 0 to 338 Pa where the amplitude $A_0 = 9.8 \times 10^{-6}$ m.

IV. The shear amplitude dependence of dissipation peaks in granular system

In Fig. S5, we plot the mechanical spectra of sand at four different shear amplitudes from 1.96×10^{-6} m to 1.28×10^{-5} m. With the increase of the amplitude, all the energy dissipation peaks and modulus minima shift slightly to low frequencies. This is because the stiffness of grain chain decreases when the chains are in loose contact under a large shear vibration. The vibration intensity Γ increases with increasing amplitude, leading the blocked grain chains become more relaxed and flexible, and so the required frequency of grain chain resonance decreases.

In order to give a further comparison on the experimental ratios of peak positions f_1/f_2 and f_1/f_3 with the theoretical predictions, the values of f_1/f_2 and f_1/f_3 are plotted at different shear amplitudes, as shown in Fig. S6. The ratios of f_2/f_1 and f_3/f_1 never change with increasing amplitude, and the experimental values are found to be quite close to the theoretical prediction lines.

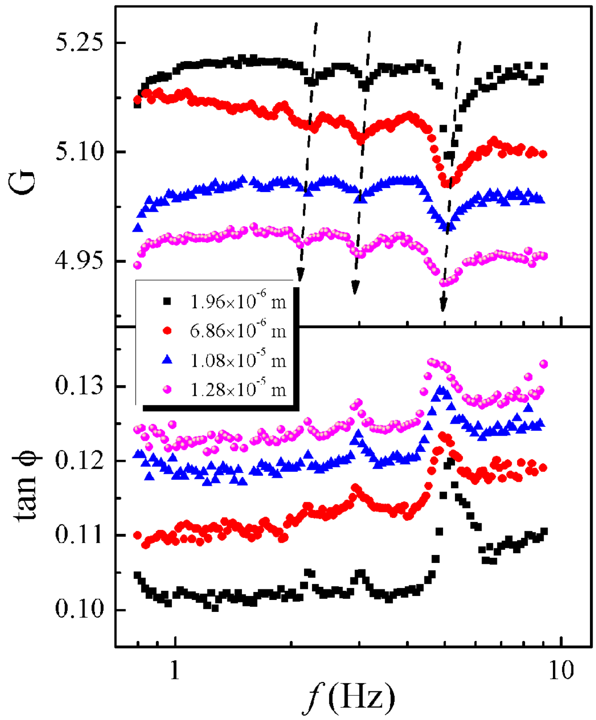

Fig. S5. Mechanical spectra (energy dissipation and modulus) of fine sand as a function of frequency, where the amplitude $A_0 = 1.96 \times 10^{-6}$ m, 6.86×10^{-6} m, 1.08×10^{-5} m, and 1.25×10^{-6} m.

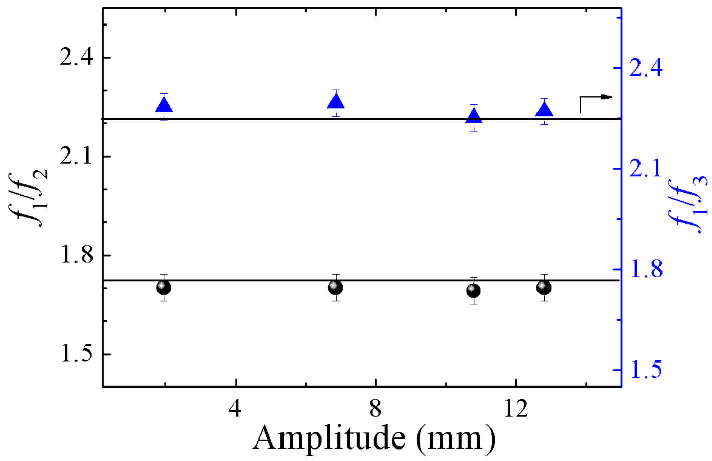

Fig. S6. Ratios of peak positions among f_1, f_2 and f_3 for P1, P2 and P3 peaks at four different shear amplitudes. The experimental values agree well with the theoretical prediction lines.