# Supplementary information for "Orientational order of one-patch colloidal particles in two dimensions"

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FIG. A A schematic drawing of neighboring one-patch particles in (a) and the development of clustering in the simulation without patch-substrate attraction in (b) and (c). (b): Patch size dependence of the development of N<sub>avg</sub> plotted against simulation steps. e<sub>b</sub> = 6.0k<sub>B</sub>T, and θ<sub>ap</sub> of the plots are 6, 18, 30, 36, 42, 60, 66, 72, 78, 84 and 90° from the bottom, respectively. The symbols of overlapping plots are as follows: (42°, △), (60°, ×), (78°, □), (84°, +), (90°, ∘). (c): Inter-patch attraction dependence of the development of N<sub>avg</sub> for θ<sub>ap</sub> = 78°. e<sub>b</sub> of the plots are 0.0, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0 and 10.0 k<sub>B</sub>T from the bottom, respectively. The top horizontal axis is for 0.0, 1.0, 2.0, and 3.0 k<sub>B</sub>T, and the bottom axis is for the others. The symbols of overlapping plots are as follows: (6.0k<sub>B</sub>T, △), (7.0k<sub>B</sub>T, ∘) and (10.0k<sub>B</sub>T, ×). In (b) and (c) larger N<sub>avg</sub> than 80 is predominantly because of the effect of boundary conditions as described in the main text.

# SIMULATION METHODS

The pairwise interaction potential between neighboring particle 1 and 2,  $u_{12}$ , is written as follows (see Fig. A (a) for the model):

$$u_{12}(\boldsymbol{n}_1, \boldsymbol{n}_2, \boldsymbol{r}_{12}) = \begin{cases} -e_{\rm b} & \text{if} \quad \boldsymbol{n}_1 \cdot \hat{\boldsymbol{r}}_{12} > \cos \theta_{\rm ap} \\ & \text{and} \quad \boldsymbol{n}_2 \cdot \hat{\boldsymbol{r}}_{12} < -\cos \theta_{\rm ap} \\ 0 & \text{otherwise} \end{cases}$$

where  $\hat{\mathbf{r}}_{12} = \mathbf{r}_{12}/r_{12}$ ,  $r_{12} = d$  and  $\mathbf{r}_{12}$  is the vector from the center of particle 1 to that of particle 2.

In a rectangular simulation box an axis of the triangular lattice of arranged particles is

parallel to y-axis, and 40 particles are arranged in y direction. For x-direction 46 particles are arranged in a zig-zag manner.

In a simulation step, a random three-dimensional rotational move is attempted for each particle. The initial orientations of particles are randomized. For efficient relaxation of clustering the range of rotation is changed in a simulation run: In the first quarter of the total steps the rotation is within  $\pm \pi$  rad, in the second  $\pm \pi/3$  rad, in the third  $\pm \pi/9$  rad and in the last  $\pm \pi/30$  rad. The total steps are from  $10^4$  to  $2 \times 10^5$  dependent on the parameters. For a set of  $e_{\rm b}$  and  $\theta_{\rm ap}$ , four independent simulation runs were carried out. As shown in Fig. A (b) and (c),  $N_{\rm avg}$  reaches its steady value in the very early stage of a simulation run. After the runs, we have never found any other steady cluster states than the ones described in the main text.

# ANALYSIS BY IMAGE PROCESSING

#### Analyses in Section 3.1.1 to 3.1.3

In a linear cluster the patches aggregates around its middle in width, we thus analyze the dark regions in an image. A microscopy image is binarized and skeletonized with a software, ImageJ (see the lines in Fig. B (b)). An obtained line is segmented by equal length measured along the line (see Fig. B (c)), and the average length for segmentation is chosen to 2.34  $\mu$ m to avoid angular fluctuation by zigzag undulation in a linear cluster. For the mapping of orientation an end-to-end or -node segment shorter than a segment of an isolated rhombic tetramer are ignored, because the orientation of a linear cluster cannot be defined for such a small cluster. Angles of segments are mapped onto their Voronoi regions by color, as shown by the color bar in Fig. B (b).

The same segments are used for the calculation of radial directional correlation,  $g_{\theta}$ . The parameters for calculating  $g_{\theta}$  is shown in Fig. B (c). For counting the number of end  $n_{\text{end}}$ , on the other hand, a skeletonized image before segmentation was used without removing short lines, for evaluating the topological property of a pattern.  $n_{\text{end}}$  gives qualitative and relative measure of the connectivity of patches; it is not directly related to the number of bonds between patches.



FIG. B Procesures of image processing and analysis with an example. (a): A microscopy image cropped from Fig. 4 (a). (b): A superposition of skeletonized dark regions, i.e. aggregating patches, in (a) onto a color-mapped image obtained from segmented lines. (c): Schematic diagram of segmentation. Thick blue curves are the lines by skeletonization, and black straight lines are the segmented ones. Angles of segment *i* and *j* and the vector connecting the centers of the segments are also shown. (d): Histgram of the area of a dark region in the sample of Fig. 7

(c). The arrows indicate the peaks of monomer, dimer and trimer from the left.

# Analysis for Fig. 7

In one-patch particles a cluster size (i.e., the number of bonded particles in a cluster) simply equals the number of aggregating patches. The area of a dark region in a microscopy image is approximately proportional to the number of patches, in particular when a linear cluster is long. In small clusters three-dimensional fluctuation of particle orientation affects the apparent area; however, quantized area distribution corresponding to monomer, dimer and trimer is observed in a histogram in Fig. B (d). The horizontal position of the third peak corresponds to the area of decomposed small clusters in Fig. 7 (f). For simplicity we estimate the size of a cluster,  $\hat{N}$ , by normalizing the area of a dark region, A, with one-third



FIG. C Particles attached to the interface of the phase-separated liquids. "L" and "W" denote lutidine- and water-rich phase respectively. (a): Microscopy images for  $d = 2.0 \ \mu\text{m}$ ,  $h_0 = 50 \ \text{nm}$  and  $\Delta T = 6.0 \text{ K}$ . Scale bar: 10.0  $\mu\text{m}$ . (b): A schematic drawing of the cross-sectional view.

of the area at the peak of trimer,  $\bar{A}_3$ :  $\hat{N} = A/(\bar{A}_3/3)$ . Because of the rough estimation, the scale of  $\hat{N}$  could deviate a few 10% from the actual number of aggregating patches, N.

#### RESULTS

#### Movies

Supplemental movies were recorded during observations of the sample in Fig. 2, 3, 4 and 7. The file name of a movie includes the corresponding figure name like "movie1\_fig2b" for Fig. 2 (b). The playing speed in each is accelerated 10 times.

#### Particles at the interface of phase-separated liquids

Figuer C (a) is an example of hemispherically-patched particles at the interface between water-rich and lutidine-rich phases. A gold patch is in lutidine-rich and bare side is in a water-rich phase. The cell surfaces are hydrophobic, the interface is thus vertical as shown in Fig. C (b).

### Other calculated patterns

Some calculated equilibrium patterns by the simulations are shown. Figure D (a) and (b) are the patterns with no patch-substrate attraction, showing the dependence on  $\theta_{ap}$  and  $e_{b}$ ,



FIG. D Calculated equilibrium patterns by simulation, showing  $e_{\rm b}$ - and  $\theta_{\rm ap}$ -dependence. Conditions and parameters of each pattern are shown in the figure. Scales are the same for (a) and (c), or (b) and (d).

respectively. Figure D (c) and (d) are the patterns with large patch-substrate attraction.