Supplementary Material for: "Motility induced changes in viscosity of suspensions of swimming microbes in extensional flows"

(Dated: April 7, 2015)

I. CALIBRATION CURVE FOR THE ACOUSTICALLY DRIVEN MICROFLUIDIC EXTENSION RHEOMETER

A rational function of the following form is chosen to fit the experimental $t^*_{1/2}$-vs-Oh data:

$$t^*_{1/2} = \frac{K_0 + K_1\,\mathrm{Oh} + K_2\,\mathrm{Oh}^2}{1 + \mathrm{Oh}} \,. \tag{1}$$

With this expression, as $\mathrm{Oh} \to 0$, $t^*_{1/2} \to K_0$, and when $\mathrm{Oh} \to \infty$, $t^*_{1/2} \to K_2\,\mathrm{Oh}$.

To fix the constants K_0 and K_2, we consider a simple toy model. The thinning of perfectly cylindrical filaments can be modeled by an inertio-viscous capillary stress balance:(Entov and Hinch, 1997; McKinley and Sridhar, 2002; Rodd et al., 2005; Tirtaatmadja et al., 2006)

$$\frac{1}{2}\,\rho \dot{R}^2 = (2X - 1)\,\frac{\gamma}{R} + \frac{6\,\eta_s\,\dot{R}}{R}\,, \tag{2}$$

where X is the ratio of the instantaneous axial tension in the liquid bridge and $2\pi\gamma R$. If X is treated as a constant, the algebro-differential equation above can be treated as a quadratic equation above for $\dot{R}$ at any R and t. The negative root of the rescaled equation,

$$\frac{dR^*}{dt^*} = -\frac{6\,\mathrm{Oh}}{R^*}\left(\sqrt{1 + C\,R^*} - 1\right), \tag{3}$$

where $C = (2X - 1)/18$ can be integrated with $R^* = 1$ as the initial condition to obtain

$$t^*_{1/2} = \frac{1}{36\,C^2}\left[4\left\{\left(\mathrm{Oh}^2 + C\right)^{3/2} - \left(\mathrm{Oh}^2 + \frac{C}{2}\right)^{3/2}\right\} + 3\,C\,\mathrm{Oh}\right]. \tag{4}$$

The equation recovers the expected results in the limits $\mathrm{Oh} \to 0$ where $t^*_{1/2}$ approaches a constant ($= (2^{3/2} - 1)/(6\sqrt{2X - 1})$, and when $\mathrm{Oh} \to \infty$ where $t^*_{1/2}$ increase linearly with Oh ($t^*_{1/2} \to 3\,\mathrm{Oh}/(2X - 1)$).

Strictly speaking, X itself depends on time; it has been shown however that, if the neck diameter is significantly smaller than the two drops at the end plates, the axial filament profile and its dynamics can be well approximated as being self-similar. As mentioned earlier, we begin taking measurements of R only after the neck is well formed with a radius about half the drop radius at the end-plate. The values of X given by similarity solutions under different conditions have been summarized by McKinley and Tripathy(McKinley and Tripathi, 2000). When inertia is important and $\mathrm{Oh} \ll 1$, $X = 0.5912$ is the value most likely to be observed in an experiment(McKinley and Tripathi, 2000; Eggers, 1997a,b) and hence $t^*_{1/2} \to 0.7135$ as

Oh $\rightarrow 0$. When viscosity is dominant on the other hand, $X = 0.7127$(McKinley and Tripathi, 2000; Papageorgiou, 1995) and $t^*_{1/2} \rightarrow 7.0522\,\mathrm{Oh}$ as Oh $\rightarrow \infty$.

These asymptotic predictions suggest therefore the values of $K_0 = 0.7135$ and $K_2 = 7.0522$ in in Eqn. (1) for the empirical calibration curve. Although the model presented above is expected to be accurate only when the axial curvature of the filament is small, we surprisingly find good agreement of the asymptotic behaviours predicted with the experimental data shown in Fig. 2 of the main text. We therefore use the values above for K_0 and K_2 in Eqn. (1) and determine $K_1 = 14.7 \pm 0.2$ by linear regression of the curve through the experimental $t^*_{1/2}$-vs-Oh data. The reasons behind this good agreement with the asymptotic trends predicted by the toy model are unknown. It is possible that this is a fortuitous result of the combination of the liquid-bridge aspect ratio, the Bond number and the range of Oh values in our experiments. A different choice of these parameters may require K_0 and K_2 also to be determined by regression.

II. CAPILLARY THINNING DATA FOR CELL SUSPENSIONS

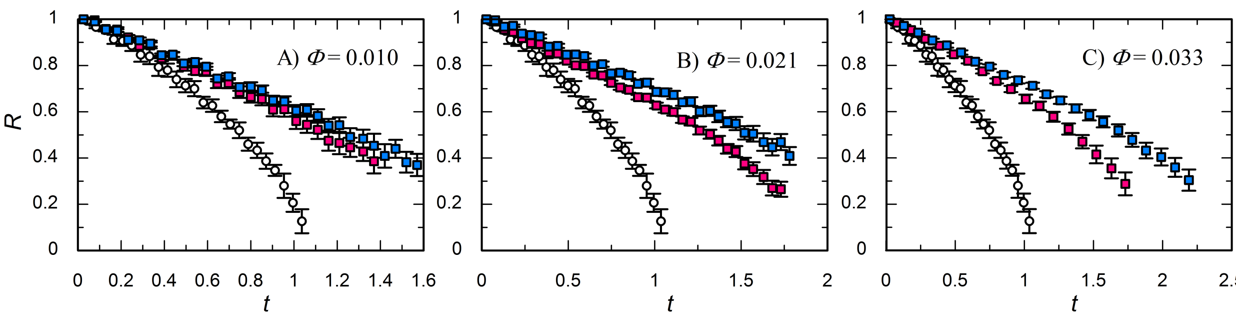

FIG. 1. Capillary thinning of liquid bridges of mouse sperm suspensions

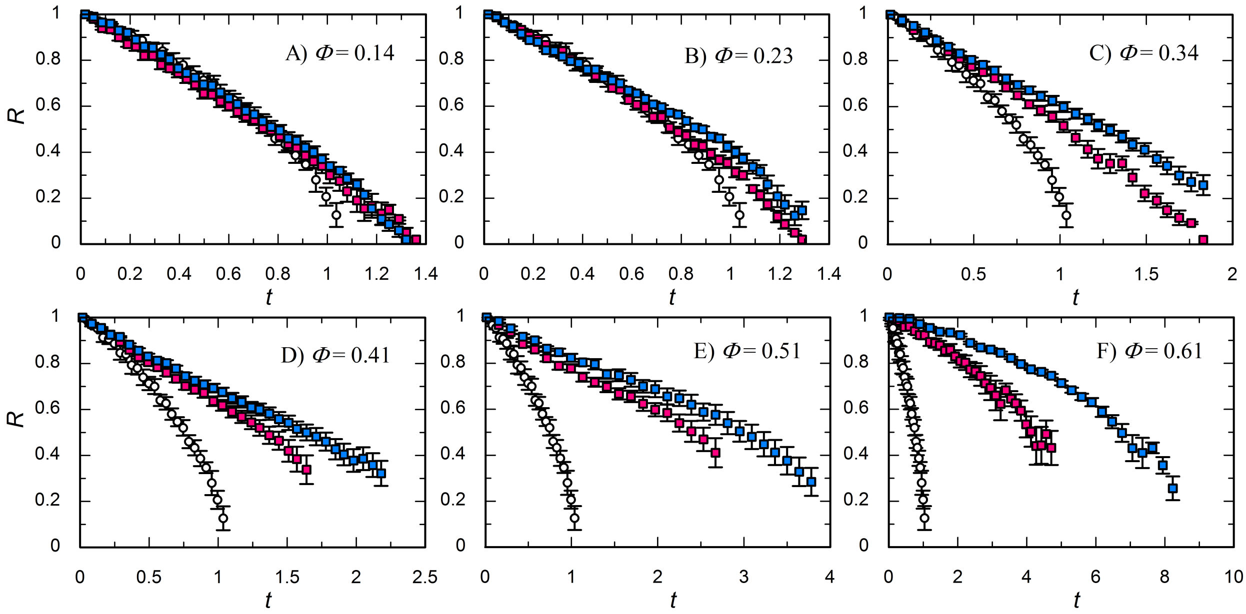

FIG. 2. Capillary thinning of liquid bridges of suspensions of *E. coli*

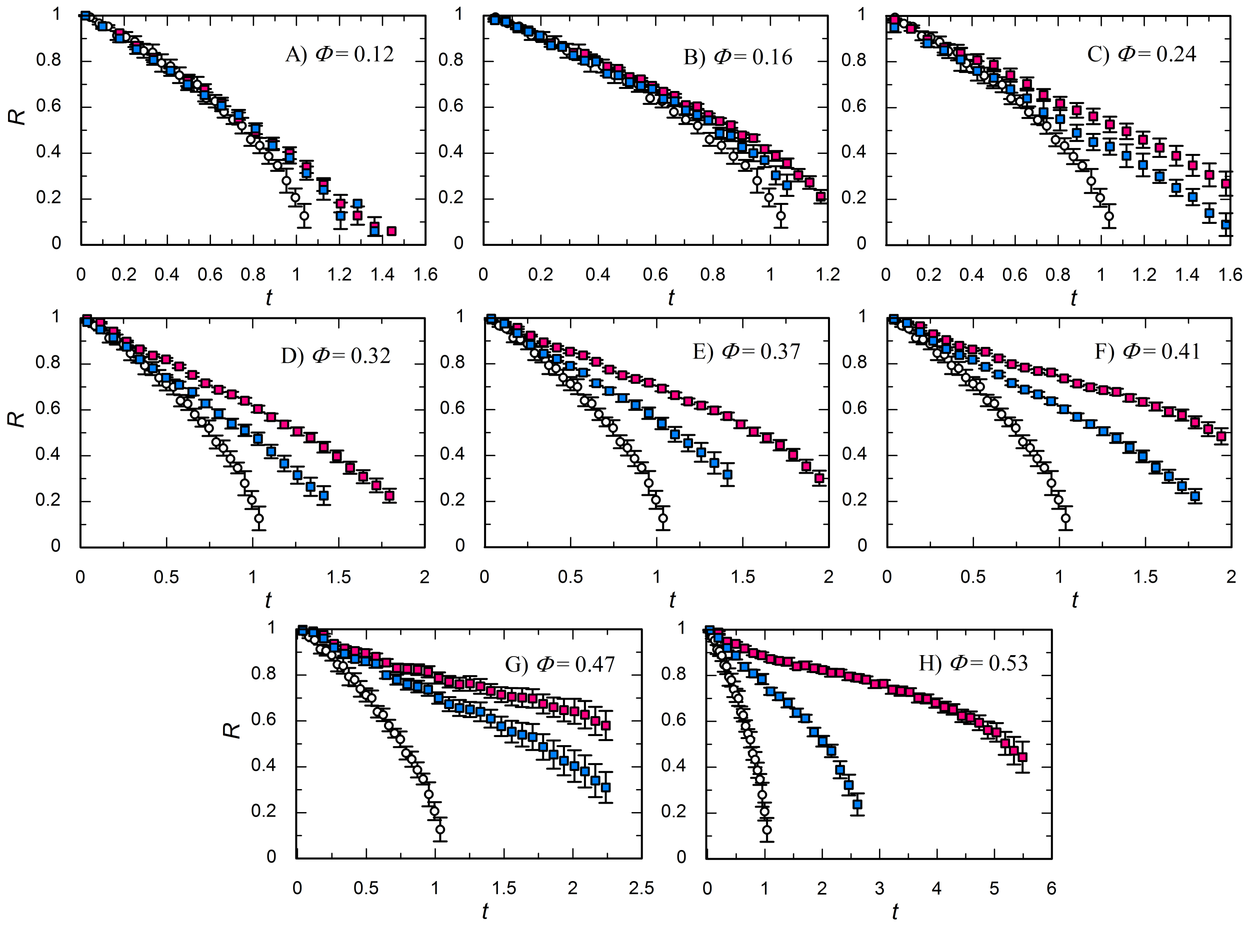

FIG. 3. Capillary thinning of liquid bridges of suspensions of *D. tertiolecta*; red, blue and white symbols correspond to live and dead cell suspensions, and pure buffer, respectively.

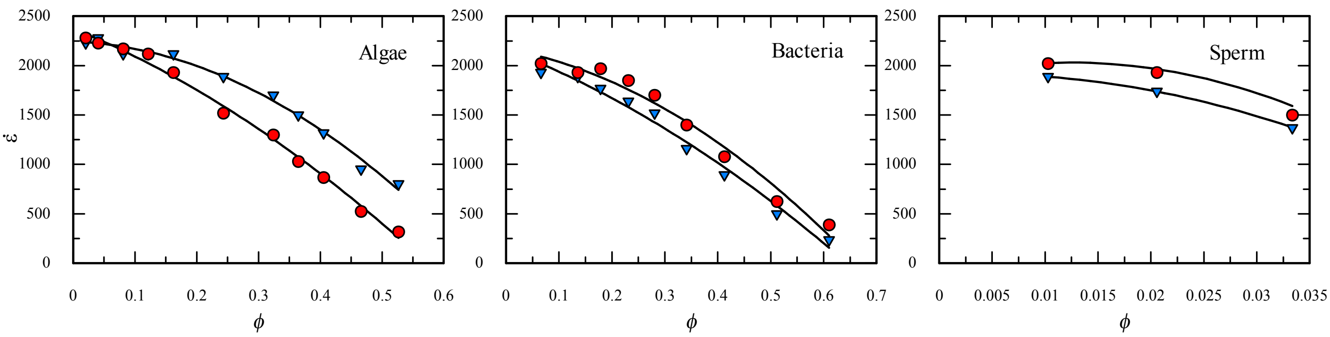

FIG. 4. Strain-rate variation with volume fraction of live (red circles) and dead (blue triangles) cell suspensions; smooth curves are cubic polynomial fits through data.

III. ADDITIONAL INFORMATION FOR THE VISCOSITY MODEL

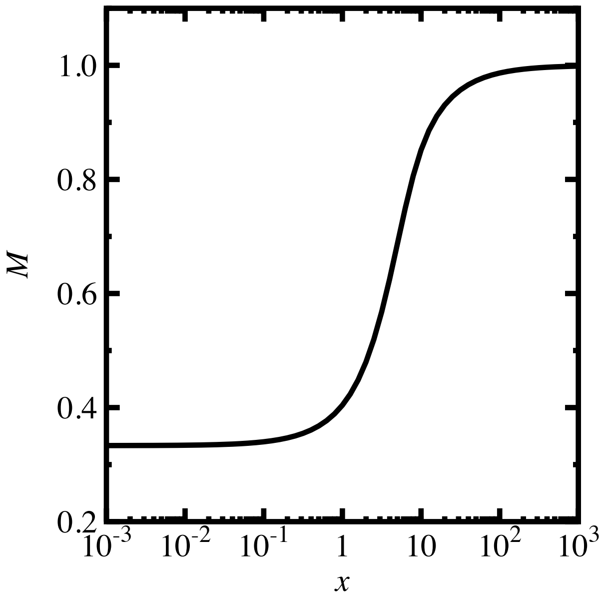

FIG. 5. The function $M(x)$

The function (Fig. 5)

$$M(x) = \frac{1}{2\,x\,D(x)} - \frac{1}{2\,x^2}\,, \tag{5}$$

where $D(x) = \exp(-x^2)\int_0^x \exp(y^2)\,dy$ is Dawson's integral. In the equation for the intrinsic extensional viscosity in the main text, $x = \sqrt{3\,\widetilde{\beta}\,\mathrm{Pe}/4}$. As $x \to 0$, $M = 1/3 + 4/45\,x^2 + O(x^4) = 1/3 + \widetilde{\beta}\mathrm{Pe}/15 + O(\mathrm{Pe}^2)$, and when $x \to \infty$, $M(x) = 1 - x^{-2} + O(x^{-4}) = 1 - 4/(3\widetilde{\beta}\mathrm{Pe}) + O(\mathrm{Pe}^{-2})$.

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