Supporting Information

Constructing Mixing π -Conjugated Bridge: A Simple and Effective Approach to Realize the Large First Hyperpolarizability in Carbon Nanotube-Based Systems

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Section I



Figure S1. The crucial transition states of cyclacene[*n*] (n = 5, 6 and 7) decorated by $-NH_2$ and $-CH=CH-NH_2$ groups.

Section II

In this work, the measurable hyper-Rayleigh scattering (HRS) first hyperpolarizabilities of the related systems were computed according to the following expression:^{1,2}

$$\beta_{\rm H R S} = \sqrt{\left\langle \beta_{ZZZ}^2 \right\rangle + \left\langle \beta_{X Z Z}^2 \right\rangle} \tag{1}$$

where the $\langle \beta_{ZZZ}^2 \rangle$ and $\langle \beta_{XZZ}^2 \rangle$ correspond to the orientational averages of the β tensor, and are presented as follows (without assuming Kleinman's conditions):

$$\left\langle \beta_{ZZZ}^{2} \right\rangle = \frac{1}{7} \sum_{\varsigma}^{x,y,z} \mathscr{P}_{\varsigma\varsigma\varsigma}^{2} + \frac{4}{35} \sum_{\varsigma\neq\eta}^{x,y,z} \mathscr{P}_{\varsigma\varsigma\eta}^{2} + \frac{2}{35} \sum_{\varsigma\neq\eta}^{x,y,z} \mathscr{P}_{\varsigma\varsigma\varsigma\varphi} \beta_{\varsigma\eta\eta} + \frac{4}{35} \sum_{\varsigma\neq\eta}^{x,y,z} \mathscr{P}_{\varsigma\varsigma\varsigma} \beta_{\varsigma\eta\eta} + \frac{4}{35} \sum_{\varsigma\neq\eta}^{x,y,z} \mathscr{P}_{\varsigma\varsigma\varsigma} \beta_{\eta\eta\varsigma} + \frac{1}{35} \sum_{\varsigma\neq\eta}^{x,y,z} \mathscr{P}_{\eta\varsigma\varsigma} \right)$$

$$+ \frac{4}{105} \sum_{\varsigma\neq\eta\neq\xi}^{x,y,z} \mathscr{P}_{\varsigma\varsigma\eta} \beta_{\eta\xi\xi} + \frac{1}{105} \sum_{\varsigma\neq\eta\neq\xi}^{x,y,z} \mathscr{P}_{\eta\varsigma\varsigma} \beta_{\eta\xi\xi}$$

$$+ \frac{4}{105} \sum_{\varsigma\neq\eta\neq\xi}^{x,y,z} \mathscr{P}_{\varsigma\varsigma\eta} \beta_{\xi\xi\eta} + \frac{2}{105} \sum_{\varsigma\neq\eta\neq\xi}^{x,y,z} \mathscr{P}_{\varsigma\eta\xi}^{2} + \frac{4}{105} \sum_{\varsigma\neq\eta\neq\xi}^{x,y,z} \mathscr{P}_{\varsigma\eta\xi} \beta_{\eta\xi\xi}$$

$$+ \frac{4}{105} \sum_{\varsigma\neq\eta\neq\xi}^{x,y,z} \mathscr{P}_{\varsigma\varsigma\eta} \beta_{\xi\xi\eta} + \frac{2}{105} \sum_{\varsigma\neq\eta\neq\xi}^{x,y,z} \mathscr{P}_{\varsigma\eta\xi}^{2} + \frac{4}{105} \sum_{\varsigma\neq\eta\neq\xi}^{x,y,z} \mathscr{P}_{\varsigma\eta\xi} \beta_{\eta\xi\xi}$$

$$+ \frac{4}{105} \sum_{\varsigma\neq\eta\neq\xi}^{x,y,z} \mathscr{P}_{\varsigma\eta\xi} \beta_{\xi\xi\eta} + \frac{2}{105} \sum_{\varsigma\neq\eta\neq\xi}^{x,y,z} \mathscr{P}_{\varsigma\eta\xi}^{2} + \frac{4}{105} \sum_{\varsigma\neq\eta\neq\xi}^{x,y,z} \mathscr{P}_{\varsigma\eta\xi} \beta_{\eta\xi\xi} \beta_{\eta\xi\xi}$$

$$\left\langle \beta_{\text{XZZ}}^{2} \right\rangle = \frac{1}{35} \sum_{\varsigma}^{x,y,z} \left\langle \beta_{\varsigma\varsigma\varsigma\varsigma}^{2} + \frac{4}{105} \sum_{\varsigma\neq\eta}^{x,y,z} \left\langle \beta_{\varsigma\varsigma\varsigma\varsigma} \right\rangle_{\varsigma\eta\eta} - \frac{2}{35} \sum_{\varsigma\neq\eta}^{x,y,z} \left\langle \beta_{\varsigma\varsigma\varsigma\varsigma} \right\rangle_{\eta\eta\varsigma} + \frac{8}{105} \sum_{\varsigma\neq\eta}^{x,y,z} \left\langle \beta_{\varsigma\varsigma\eta}^{2} + \frac{3}{35} \sum_{\varsigma\neq\eta}^{x,y,z} \left\langle \beta_{\varsigma\eta\eta}^{2} - \frac{2}{35} \sum_{\varsigma\neq\eta}^{x,y,z} \left\langle \beta_{\varsigma\varsigma\eta} \right\rangle_{\eta\varsigma\varsigma} + \frac{1}{35} \sum_{\varsigma\neq\eta\neq\xi}^{x,y,z} \left\langle \beta_{\varsigma\eta\eta} \right\rangle_{\varsigma\varsigma\xi\varsigma} - \frac{2}{105} \sum_{\varsigma\neq\eta\neq\xi}^{x,y,z} \left\langle \beta_{\varsigma\varsigma\eta} \right\rangle_{\eta\varsigma\varsigma} - \frac{2}{105} \sum_{\varsigma\neq\eta\neq\xi}^{x,y,z} \left\langle \beta_{\varsigma\varsigma\eta} \right\rangle_{\eta\xi\xi} + \frac{2}{35} \sum_{\varsigma\neq\eta\neq\xi}^{x,y,z} \left\langle \beta_{\varsigma\eta\xi}^{2} - \frac{2}{105} \sum_{\varsigma\neq\eta\neq\xi}^{x,y,z} \left\langle \beta_{\varsigma\eta\xi} \right\rangle_{\eta\varsigma\xi} \right\rangle_{\eta\xi\xi}$$

$$(3)$$

Here, the correlative components of static fist hyperpolarizability tensor in these equations are obtained by using the coupled perturbed Hartree-Fock (CPHF) method with the basis set 6-31G(d). All the computations were carried out by using the GAUSSIAN 09 program package.

As shown in the following Figures S2, S3 and S4, it was found that those findings based on the computed results of β_0 in the text can also be supported by their corresponding computed β_{HRS} results.

References:

(1) R. Bersohn, Y. H. Pao, H. L. Frisch, J Chem Phys 1966, 45, 3184.

(2) S. N. Labidi, M. B. Kanoun, M. D. Wergifosse, B. Champagne, *Inter. J. of Quantum Chem.* 2011, **111**, 1583.



Figure S2. The HRS first hyperpolarizabilities (β_{HRS}) of (**a**) cyclacene[6]–CH=CH–NH₂, (**b**) cyclacene[6]–(CH=CH)₂–NH₂ and (**c**) cyclacene[6]–(CH=CH–NH₂)₂.



Figure S3. The relationship of the HRS first hyperpolarizability (β_{HRS}) versus *x* values for both CNT₂-(CH=CH)_{*x*}-NH₂ (*x* = 1, 2, 3, 6 and 9) and CNT_{2+*x*}-NH₂ (*x* = 1, 3, 4, 6 and 8) series.



Figure S4. Dependence of the HRS first hyperpolarizability (β_{HRS}) value on the length of the $-(\text{CH}=\text{CH})_x$ - chain (*x*) in CNT_{Γ} -(CH=CH)_x-NH₂ (l + x = 8, x = 0, 2, 4, 6 and 8).